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Abstract: The number of publications in the area of Cutting and Packing (C & P) has increased considerably over the last two decades. The typology of C & P problems introduced by Dyckhoff (1990) initially provided an excellent instrument for the organisation and categorisation of existing and new literature. However, over the years also some deficiencies of this typology became evident, which created problems in dealing with recent developments and prevented it from being accepted more generally. In this paper, the authors present an improved typology, which is partially based on Dyckhoff's original ideas, but introduces new categorisation criteria, which define problem categories different from those of Dyckhoff. Furthermore, a new, consistent system of names is suggested for these problem categories. Finally, the practicability of the new scheme is demonstrated by using it as a basis for a categorisation of the C & P literature from the years between 1995 and 2004.

Key Words: Cutting, Packing, Typology

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1 Introduction

A typology is a systematic organisation of objects into homogenous categories on the basis of a given set of characterising criteria. It is practically oriented and meant to deal with "important" real objects in the first place while hypothetical and/or "less important" real objects might be neglected. A typology resembles a classification, however, with respect to the above-mentioned focus, unlike the latter, it may not be complete (i. e. not all properties of a criterion may be considered explicitly), and it may be "fuzzy" (i. e. the categories may not always be defined precisely and properly distinguished from each other).

A typology provides a concise view of "relevant", "important" objects and, thus, prepares the ground for practically oriented research. Furthermore, it helps to unify definitions and notations and by doing so, facilitates communication between researchers in the field. Finally, if publications are categorised according to an existing typology, it will provide a faster access to the relevant literature.

A typology of OR problems, in particular, provides the basis for a structural analysis of the underlying problem types, the identification and definition of standard problems, the development of models and algorithms, problem generators etc. For the area of cutting and packing (C & P), Dyckhoff (1990) has introduced such a typology, which, unfortunately, has not always found satisfactory international acceptance. Furthermore, almost fifteen years since its initial publication, it became obvious that Dyckhoff's typology is insufficient with respect to the inclusion of current developments. Therefore, the authors decided to present a new, improved typology here, providing a consistent system of problem types which allows for a complete categorisation of all known C & P problems and the corresponding literature. Furthermore, it helps to identify "blank spots", i.e. areas, in which none or only very little research has been done. Finally, the suggested typology is also open to new problem types to be introduced in the future.

For the problem types included in the typology, a straightforward system of notations is introduced. Wherever possible, the problem categories are named in accordance with existing notations. By doing so, the authors do not only intend to avoid unnecessary misinterpretations, or even confusion, but also to improve the degree of acceptance among researchers in the field. The system of notations will also provide the basis for a system of abbreviations which can be used in a database of C & P literature for a simple but precise characterisation. The suggested abbreviations may also become part of a more complex coding scheme of C & P problems.

The remaining part of this paper is organised as follows: In section 2, the general structure of C & P problems will be presented, not only because the typology to be introduced here addresses C & P problems, but also because the criteria for the definition of the respective problem categories are closely related to elements of this structure. In section 3, Dyckhoff's typology will be discussed, and, in particular, some severe drawbacks are pointed out which necessitate the development of a new, improved typology. The new typology is outlined in section 4. In section 5, criteria and corresponding properties used in the new typology for the definition of problem types are introduced. As far as these criteria have also been used by Dyckhoff, they were complemented by additional, potentially important properties. Then (section 6), in a first step, two of the criteria, "type of assignment" and "assortment of small objects”, are used to define basic problem types. The additional application of a third criterion, “assortment of large objects” leads to intermediate problem types. Finally, the application of the criteria “dimensionality” and “shape of small items” provides refined problem types. In section 7 it is demonstrated how well-known problems discussed in the literature fit into the suggested typology. Furthermore, its usefulness and practicability is demonstrated by categorising the literature on C & P problems from the last decade (1995-2004) accordingly. An analysis of recent trends in C & P research, based on the categorisation, is given in section 8. The paper concludes with an outlook on future work (section 9), which particularly sketches a potential, more refined scheme for the categorisation of (publications on) C & P problems.

2 Structure of Cutting and Packing Problems

Cutting and packing problems have an identical structure in common which can be summarised as follows:

Given are two sets of elements, namely

- a set of large objects (input, supply) and
- a set of small items (output, demand),

which are defined exhaustively in one, two, three or an even larger number (n) of geometric dimensions. Select some or all small items, group them into one or more subsets and assign each of the resulting subsets to one of the large objects such that the *geometric condition* holds, i. e. the small items of each subset have to be laid out on the corresponding large object such that

- all small items of the subset lie entirely within the large object and
- the small items do not overlap,

and a given (single-dimensional or multi-dimensional) objective function is optimised. We note that a solution of the problem may result in using some or all large objects, and some or all small items, respectively.

Formally, five sub-problems can be distinguished, which, of course, have to be solved simultaneously in order to achieve a “global” optimum:

- selection problem regarding the large objects;
- selection problem regarding the small items;
- grouping problem regarding the selected small items;
- allocation problem regarding the assignment of the subsets of the small items to the large objects;
- layout problem regarding the arrangement of the small items on each of the selected large objects with respect to the geometric condition.

Special (types of) C & P problems are characterised by additional properties. In particular, they may turn out to be degenerated in the sense that they do not include all of the above-mentioned sub-problems.

3 Dyckhoff's Typology

Fig. 1 summarises the main features of Dyckhoff's typology. The author introduces four criteria (characteristics) according to which C & P problems are categorised.

1. Dimensionality
   (1) one-dimensional
   (2) two-dimensional
   (3) three-dimensional
   (N) N-dimensional with N > 3
2. Kind of assignment
   (B) all objects and a selection of items
   (V) a selection of objects and all items
3. Assortment of large objects
   (O) one object
   (I) identical figure
   (D) different figures
4. Assortment of small items
   (F) few items (of different figures)
   (M) many items of many different figures
   (R) many items of relatively few different (non-congruent) figures
   (C) congruent figures

Fig. 1: Categorisation criteria, main types, and coding scheme of Dyckhoff's typology (cf. Dyckhoff 1990, p. 154)

Criterion 1, dimensionality, captures the minimal number (1, 2, 3, n > 3) of geometric dimensions which are necessary to describe the required layouts (patterns) completely. Regarding criterion 2, the kind of assignment of small items to large objects, Dyckhoff distinguishes two cases, indicated by B and V. B stands for the German "*Beladeproblem*", meaning that all large objects have to be used. A selection of small items has to be assigned to the large objects. V (for the German "*Verladeproblem*") characterises a situation in which all small items have to be assigned to a selection of large objects. Criterion 3 represents the assortment of the large objects. O stands for one large object, I for several but identical large objects, and D for several different large objects. Criterion 4 likewise characterises the assortment of the small items. Dyckhoff distinguishes assortments consisting of few items (F), many items of many different figures (M), many items of relatively few different figures (R), and congruent figures (C; we will be using the notion "shape" here, instead of "figure").

When it was initially published, Dyckhoff's work represented a milestone in C & P research as it highlighted the common underlying structure of cutting problems on one hand and packing problems on the other. By doing so, it supported the integration and cross-fertilisation of two largely separated research areas. Unfortunately, on an international level his typology has not always been accepted as widely as was desirable, most probably due to the fact that the provided coding scheme was not self-explanatory from the view point of an international (English-

speaking) community of researchers. This becomes particularly evident for the problem types B and V.
Obviously, this deficiency could be overcome by introducing more appropriate (English) names for the respective problem categories / types. However, some drawbacks have to be taken more seriously, which became evident in the light of some recent developments in the field of C & P.

- Not necessarily all C & P problems (in the narrow sense) can be assigned uniquely to problem types

  In Dyckhoff's paper already, a well-known standard problem, the *Vehicle Loading Problem*, has been coded both as 1/V/I/F and 1/V/I/M (Dyckhoff 1990, p. 155). F characterises a situation in which few items of different shapes (i.e. items which are different with respect to shape, size and/or orientation) are to be assigned. M represents a situation with many items of many different shapes. Apart from the fact that it is definitely desirable to have only one option available for the assignment of a well-known standard problem like the Vehicle Loading Problem, it does not become clear how this differentiation will provide a set of problem categories which is more homogeneous (with respect to model building and the development of algorithms) than a single type, which includes both categories. (It is interesting to note that problem category F only appears once in Dyckhoff's examples of combined types (1990, p. 155). One may take this as an additional indicator that this kind of differentiation might not prove very promising in the end.)

- Dyckhoff's typology is partially inconsistent; its application might have confusing results

  In the literature, a particular two-dimensional packing problem, the so called "*Strip-Packing Problem*", namely the packing of a set of small items (often rectangles) of different sizes into a single rectangle with fixed width and minimal (variable) length (Jacobs 1996, Martello, Monaci & Vigo 2003), has attracted considerable attention. Most probably, this problem would be coded both by researchers and by practitioners as 2/V/O/M, while in Dyckhoff's typology it has been assigned the notation 2/V/D/M. Dyckhoff justifies this notation – rather artificially from our point of view – by saying that this problem "… is equivalent to an assortment selection problem where the stock is given by an infinite number of objects of this width and of all possible lengths and where only one object has to be chosen from stock, namely that of minimal length" (1990, p. 155). What makes Dyckhoff's characterisation of the problem even more confusing is the fact that he calls it a *Two-Dimensional Bin Packing Problem* (Dyckhoff 1990, p. 155). Not only in our understanding (also see Lodi, Martello & Monaci 2002, Miyazawa & Wakabayashi 2003, Faroe, Pisinger & Zachariasen 2003) it would have been more obvious to reserve this name for the natural extension of the *Classic (One-Dimensional) Bin Packing Problem*, i.e. the packing of a set of small items of different sizes into a minimum number of rectangles (large objects) of identical size. The notation becomes even more questionable for two-dimensional problems where both width and length are variables, and, likewise, for three-dimensional problems, where width, length and / or height are variables. We conclude that – in order to avoid confusion about the contents of problem notations and definitions – it is advisable to distinguish problem types with respect to fixed and with respect to variable dimensions.

- Application of Dyckhoff's typology does not necessarily result in homogeneous problem categories

  Gradišar, Resinovič & Kljajić (2002) notice that in the case of one-dimensional cutting, where a large number of small items of relatively few different shapes has to be produced from standard material in unlimited supply, two situations should be distinguished if the standard material comes in different sizes. In the first situation, the large objects can be separated into to a few groups of identical size, while in the second situation all large objects are entirely different. In Dyckhoff's coding scheme both situations would be included in the problem type 1/V/D/R. Gradišar, Resinovič & Kljajić (2002, p. 1208) now argue that the inclusion in the same problem category is not very useful because the two situations require different solution approaches, namely a pattern-oriented approach for cutting problems with few groups of identical large objects and an item-oriented approach for such problems with entirely different large objects. We just would like to add that the same can be observed for problems of higher dimensions and for packing problems, as well. Therefore, we conclude that, in order to develop categories of homogeneous problems, the properties (main types) of Dyckhoff's third criterion, the assortment of large objects, should be further differentiated.

The latter aspect turns out to represent the most severe limitation of Dyckhoff's typology, as it generally questions whether one of the central goals of the introduction of a problem typology is achieved, namely to provide a homogeneous basis for the development of models and algorithms.

4 Outline of the New Typology and Overview of Problem Categories

Problem types can generally be defined as elementary types or combined types. Types which can be used as a basis for the development of models, algorithms, and problem generators, and for the categorisation of literature must be relatively homogeneous. Consequently, it is very likely that they must be introduced as combined types, which stem from the subsequent or simultaneous application of different categorisation criteria. Five criteria will be used here for the definition of combined problem types of C & P problems, namely "dimensionality", "kind of assignment", "assortment of large objects", "assortment of small items", and "shape of the small items".

It goes without saying that each typology of C & P problems should offer an option to characterise a given problem with respect to the number of problem-relevant dimensions. Consequently, this criterion will be adopted directly from Dyckhoff's typology.

Dyckhoff's criterion "kind of assignment" has proven to be useful in distinguishing between different kinds of C & P problems in the past. Therefore, it will also be used here; however, in order to avoid the German notations "Verladeproblem" and

"Beladeproblem", the authors will refer to the corresponding problem categories / elementary types – in a general sense - as "input minimisation" and "output maximisation", respectively.

On the other hand, in order to overcome the above-sketched limitations of Dyckhoff's typology, the two criteria "assortment of large objects" and "assortment of small items" will be redefined and/or supplemented with new properties.

Unlike in Dyckhoff's typology, the two criteria "kind of assignment" and "assortment of small items" will not be taken for the definition of two different fields in a classification scheme for C & P problems. Instead, they will be used in combination in order to define ***basic problem types***. These basic problem types (which already represent combined types in the sense of Dyckhoff; see Dyckhoff 1990, p. 154), provide the core objects for the introduction of a new, more widely accepted nomenclature. Existing names have been adopted as far as possible, in particular, wherever there was no or only a small probability that their use would result in misinterpretations of their contents.

The subsequent application of the criterion "assortment of large objects" will provide ***intermediate problem types***. Further characterisation with respect to the number of problem relevant dimensions ("dimensionality") and - in the case of problems of two and more dimensions – with respect to the "shape of small items" will provide ***refined*** (combined) ***problem types***. The name of each of these refined types consists of the name of the underlying intermediate type and one or two additional adjectives which indicate the respective properties. Refined problem types will be used for the categorisation of publications, here.

An instance of a specific refined problem type will exhibit all the properties which have been used for the definition of the respective category and probably additional constraints and / or characteristics. A problem instance which only exhibits the defining properties, but no additional constraints or characteristics could be interpreted as being (an instance of) a **(first-level)** ***standard problem (type)***. **First-level, non-standard problems** (problem types) are characterised by the properties defining the respective problem category and additional constraints and / or characteristics. Of course, in particular when considered in scientific research, they may represent well-known standard problems as well. In such case, in order to distinguish them from the previously mentioned ones, we would call them **second-level standard problems** whenever necessary. The identification of standard problems (types) represents a major goal of our typology. These problem types provide the basis of scientific research, in particular for the development of (standard) models, algorithms, and problem generators.

Fig. 2 gives an overview of the previously introduced types and their relationships. In this paper, we concentrate on "***pure***" ***C & P problem types*** in the above-defined sense (see section 2), i.e. problems in which the solution consists of information on the set of patterns according to which the (selected) small items have to be cut from / packed into (a subset of) the large objects and the corresponding objective function value. The inclusion of additional aspects which extend the view of the planning problem beyond the core of cutting or packing will give rise to an ***extended problem type*** or a ***problem extension***.

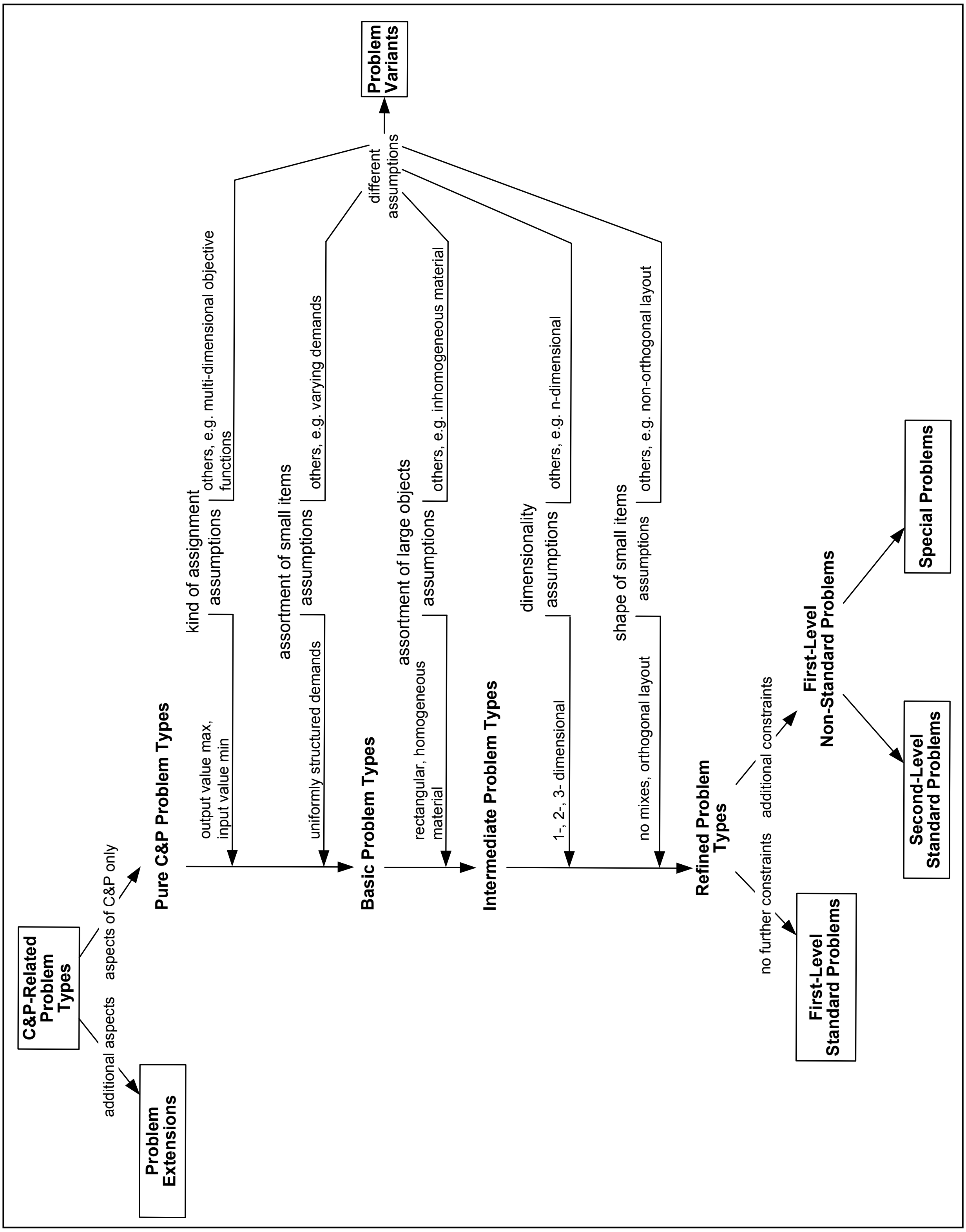

Fig. 2: *Overview of problem types related to C & P problems*

Apart from information on the patterns, a solution to a problem of this kind includes additional information on other problem-relevant aspects such as the number of different patterns (as in the pattern minimisation problem; cf. Vanderbeck 2000), processing sequences (as in the pattern sequencing problem; cf. Foerster & Wäscher 1998, Yanasse 1997, Yuen 1995, Yuen & Richardson 1995) or lot-sizes (cf. Nonås & Thorstenson 2000). We would like to point out, however, that the "C & P-related part" of problems of this type can be categorised in the same way as "pure" problems.

When defining basic, intermediate and refined problem types, it will be necessary to introduce certain assumptions. Some of these assumptions are related to general properties of the problem and restrict the view to single-objective, single-period and deterministic problems. Replacing the assumptions by different ones, leads to problem types which will be considered as ***problem (type) variants***, here. Problems with multiple objectives (cf. Wäscher 1990), stochastic problems (in which, e.g., the sizes of the large objects are random variables; cf. Das & Ghosh 2003), fuzzy problems (where the costs of the large objects are fuzzy coefficients; cf. Katagiri, Ishii & Sakawa 2004), or on-line problems (in which, e.g., parcels arrive one after another at a packing station and a decision has to be made immediately on their arrival where they should be packed into a container; cf. Hemminki, Leipälä & Nevalainen 1998, Abdou & Elmarsry 1999) belong to this category. A second class of assumptions also leading to problem variants arises from C & P-specific aspects. They will be explained in greater detail later.

5 (Modified) Criteria for the Definition of Problem Types

5.1 Dimensionality

We distinguish between ***one-, two-,*** and ***three-dimensional*** problems. In the literature, occasionally, also problems with more than three geometric dimensions are considered (e.g. Lins, Lins & Morabito 2002). Problems of this type ($n > 3$) are looked upon as variants, here.

5.2 Kind of Assignment

Again, as in Dyckhoff (1990), we introduce two basic situations, of which we prefer to speak of ***output (value) maximisation*** and ***input (value) minimisation***, respectively.

- output (value) maximisation

  In the case of output (value) maximisation, a set of small items has to be assigned to a given set of large objects. The set of large objects is not sufficient to accommodate all the small items. All large objects are to be used (in other words: there is no selection problem regarding the large objects), to which a selection (a subset) of the small items of maximal value has to be assigned.

- input (value) minimisation

  Again, a given set of small items is to be assigned to a set of large objects. Unlike before, in the case of input (value) minimisation the set of large objects is sufficient to accommodate all small items. All small items are to be assigned to a selection (a subset) of the large object(s) of minimal “value”. There is no selection problem regarding the small items.

Here, “output (value) maximisation” and “input (value) minimisation” are used in a general, non-specific manner. When treating specific problems, the “value” of objects / items has to be defined more precisely and may be represented by costs, revenues, or material quantities. Often, the value of the objects / items can be assumed to be directly proportional to their size such that the objective function considers length (one-dimensional problems), area (two-dimensional problems), or volume (three-dimensional problems) maximisation (output) or minimisation (input). In such cases, it might also be possible to translate both “output (value) maximisation” and “input (value) minimisation” into “waste minimisation”, i.e. the minimisation of the total size of unused parts of the (selected) large objects. In the environment of cutting problems often the term “trim-loss minimisation” is used.

We would also like to point out that - in order to define basic problem types – only these two situations will be considered here. Of course, problems encountered in practice and / or discussed in the literature may be characterised by the fact that a selection problem exists with respect to both large objects and small items. This requires an extended objective function which combines revenues and costs (“profit maximisation”). Also situations exist in which more than one objective function may have to be taken into account. Again, problems of this type will be considered as problem variants here.

5.3 Assortment of Small Items

With respect to the assortment of the small items we distinguish three cases, namely ***identical small items***, a ***weakly heterogeneous assortment of small items***, and a ***strongly heterogeneous assortment of small items*** (cf. Fig. 3):

- identical small items

  Regarding their problem-relevant dimensions (i.e. their extension in the problem-relevant number of dimensions “length”, “width”, and “height”), all items are of the same shape and size. In the output maximisation case, it can be assumed that the (single) item type has an unlimited demand. This problem category is identical with Dyckhoff’s elementary type C (“congruent shapes”; cf. Dyckhoff 1990, p. 154).

- weakly heterogeneous assortment

  The small items can be grouped into relatively few classes (in relation to the total number of items), for which the items are identical with respect to shape and size. By definition, small items of identical shape and size which require different orientations are treated as different kinds of items. The demand of each item type is relatively large, and may or may not be limited by an upper bound. This category corresponds to Dyckhoff’s elementary type R (“many items of relatively few different (non-congruent) shapes”; cf. Dyckhoff 1990, p. 154).

- strongly heterogeneous assortment

  The set of small items is characterised by the fact that only very few elements are of identical shape and size. If that occurs, the items are treated as individual elements. Consequently, the demand of each item is equal to one. This category includes Dyckhoff’s elementary types M (“many items of many different shapes”) and F (“few items (of different shapes)”; cf. Dyckhoff 1990, p. 154).

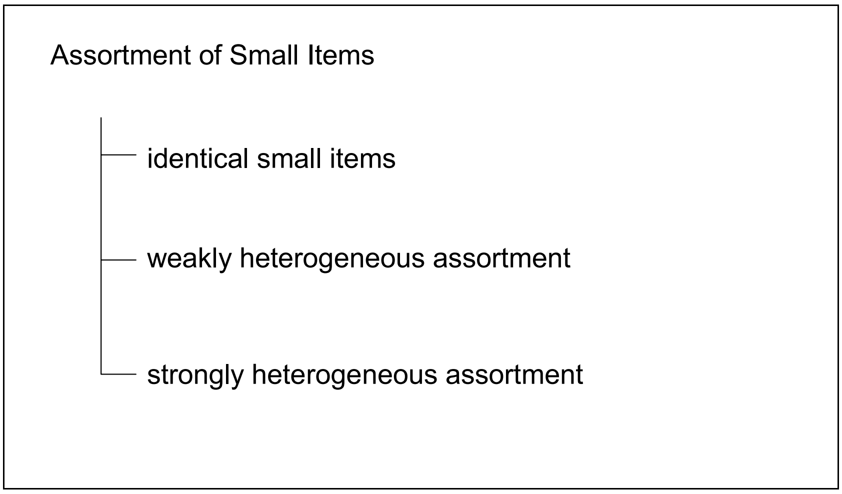

Fig. 3: *Cases to be distinguished with regard to the assortment of small items*

For the definition of standard problems we assume that the set of small items is uniformly structured, i.e. that it does not contain items with large demands and others with small demands. A problem with strongly varying demands (cf. Riehme, Scheithauer, Terno 1996) will be considered as a variant, here.

5.4 Assortment of Large Objects

With respect to the assortment of the large objects we introduce the following cases (cf. Fig. 4):

- one large object

  In this case the set of large objects consists of a single element. The extension of the large object may be fixed in all problem-relevant dimensions (“***all dimensions fixed***”), or its extension may be variable in one or more dimensions (“***one or more variable dimensions***”). The first category is identical with Dyckhoff’s type O, while the second category represents an extension of Dyckhoff’s set of elementary types (cf. Dyckhoff 1990, p. 154).

- several large objects

  With respect to the kind of problems which are described in the literature, in the case of several large objects it does not appear necessary to distinguish between fixed and variable dimensions; only fixed dimensions will be considered. In analogy to the categories which have been introduced for the assortment of the small items, we distinguish between ***identical large objects***, a ***weakly*** and a ***strongly heterogeneous assortment of large objects***. By doing so, we again extend Dyckhoff's typology, who only identifies large objects with identical (type I) and different shapes (type D).

For the definition of basic problem types we assume that - in the two- and three-dimensional case - all large objects are of rectangular shape (rectangles, cuboids) and consist of homogeneous material. Non-rectangular large objects (e.g. circular objects such as discs) and / or non-homogeneous large objects (e.g. stock material including defects) give rise to problem variants, again.

Assortment of Large Objects

- one large object
  - all dimensions fixed
  - one or more variable dimensions
- several large objects (all dimensions fixed)
  - identical large objects
  - weakly heterogeneous assortment
  - strongly heterogeneous assortment

Fig. 4: *Cases to be distinguished with regard to the assortment of large objects*

5.5 Shape of Small Items

In the case of two- and three-dimensional problems, for the definition of refined problem types we further distinguish between ***regular small items*** (rectangles, circles, boxes, cylinders, balls, etc) and ***irregular*** (also called: ***non-regular***) ones. In the two-dimensional case, the former are sometimes further distinguished into ***rectangular items***, ***circular items***, and others.

In accordance with what is usually considered in the literature, we assume that rectangular items are to be laid out orthogonally. Furthermore, the set of small items either consists of regular or irregular elements. Problems which allow for non-orthogonal layouts and / or mixes of regular and irregular small items will be looked upon as problem variants again.

6 Basic, Intermediate and Refined Problem Types

6.1 Basic Problem Types

Basic types of C & P problems are developed by combination of the two criteria “type of assignment” and “assortment of small items”. Fig. 5 depicts the relevant combinations and the corresponding basic problem types.

In the following sections these problem types will be characterised in greater detail.

6.1.1 Output Maximisation Types

Problems of the output maximisation type have in common that the large objects are only supplied in limited quantities which do not allow for accommodating all small items. As the value of the accommodated items has to be maximised, all large objects will be used. In other words, generally there is a selection problem regarding the small items, but none regarding the large objects.

According to Fig. 5, we distinguish the following (basic, output maximisation) problem types:

- **Identical Item Packing Problem**

  This problem category consists of the assignment of the largest possible number of identical small items to a given, limited set of large objects. We note that, due to the fact that all the small items are identical, there is in fact no real selection problem regarding the small items, and, furthermore, neither a grouping nor an allocation problem occurs. In other words, the general structure of C & P problems (cf. sec. 2) is reduced to a layout problem regarding the arrangement of the (identical) small items on each of the large objects with respect to the geometric condition.

- **Placement Problem**

  In the literature, problems of this category are known under many different names. In order to avoid additional confusion, here we have introduced a somewhat more neutral notion. Here, the term “Placement Problem” defines a problem category in which a weakly heterogeneous assortment of small items has to be assigned to a given, limited set of large objects. The value or the total size (as an auxiliary objective) of the accommodated small objects has to be maximised, or, alternatively, the corresponding waste has to be minimised.

- **Knapsack Problem**

  According to our interpretation, the Knapsack Problem represents a problem category which is characterised by a strongly heterogeneous assortment of small items which have to be allocated to a given set of large objects. Again, the availability of the large objects is limited such that not all small items can be accommodated. The value of the accommodated items is to be maximised.

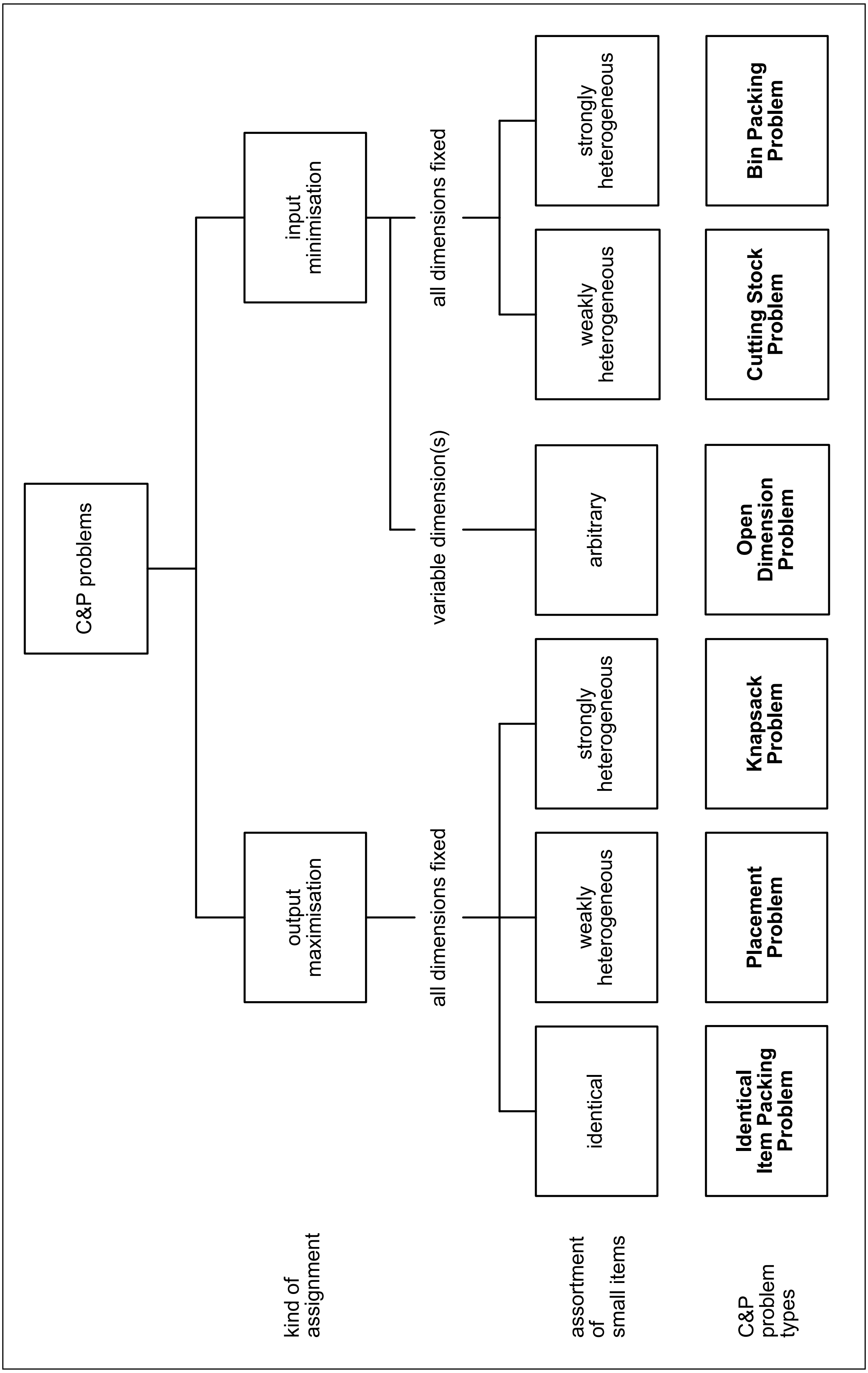

Fig. 5: *Basic problem types*

6.1.2 Input Minimisation Types

Problems of the input minimisation type are characterised by the fact that the supply of the large objects is large enough to accommodate all small items. Their demands have to be satisfied completely, so that no selection problem regarding the small items exists. The value of the large objects necessary to accommodate all small items has to be minimised.

- **Open Dimension Problem**

  The Open Dimension Problem defines a problem category in which the set of small items has to be accommodated completely by one large object or several large objects. The large objects are given, but their extension in at least one dimension can be considered as a variable. In other words, this problem involves a decision on fixing the extension(s) in the variable dimension(s) of the large objects. Only the part(s) of the large object(s) necessary to accommodate the items completely represents input within the meaning of the general structure of C & P problems (cf. section 2). The value of the input (or a corresponding auxiliary measure like length, size, or volume) is to be minimised. Obviously, problems of this type are only possible in two and more dimensions. For the definition of basic problem types we restrict ourselves to large objects which – before and after having fixed their extension(s) in the variable dimension(s) – are rectangles (two-dimensional problems) or cuboids (three-dimensional problems). By doing so, in particular those problems are excluded from our analysis in which the small items have to be enclosed in non-rectangular large objects of minimal size (e.g. when circles have to be packed into another circle of minimal radius; cf. Birgin, Martinez & Ronconi 2005, pp. 27 ff.), or in which the density of the packing has to be maximised (e.g. as in the case of two-dimensional lattice packing; cf. Stoyan & Patsuk 2000).

- **Cutting Stock Problem**

  Problems of this category require that a weakly heterogeneous assortment of small items is completely allocated to a selection of large objects of minimal value, number, or total size. The extension of the large objects is fixed in all dimensions. We point out that we do not make any assumptions on the assortment of the large objects. It may consist of identical objects, but it could also be a weakly or strongly heterogeneous assortment.

- **Bin Packing Problem**

  In contrast to the previously described problem category, this one is characterised by a strongly heterogeneous assortment of small items. Again, the items have to be assigned to a set of identical large objects, a weakly heterogeneous or strongly heterogeneous assortment of large objects. The value, number, or total size of the necessary large objects (or another corresponding auxiliary objective) has to be minimised.

6.2 Intermediate Problem Types

In order to define more homogeneous problem types the above-developed basic problem types are structured further into intermediate problem types. This is achieved by taking into account the assortment of the large objects as an additional differentiating criterion. Fig. 6 and Fig. 7 depict the intermediate problem types related to output maximisation, Fig. 8 and Fig. 9 the intermediate types related to input minimisation. Fig. 6 –9 also present our suggestions for naming these types.

We note at this stage that – due to its simple problem structure – it is not necessary to further differentiate the Identical Item Packing Problem. In order to solve a problem of this type, it can be split into a set of independent sub-problems where each sub-problem is related to a particular large object (or a particular type of large objects, if the large objects are at least partially identical) to which the largest possible number of small items has to be assigned. Consequently, in this typology the name Identical Item Packing Problem will also be reserved for a (combined) problem type in which the largest possible number of identical small items has to be assigned to a single given large object. Furthermore, with respect to the fact that the existing literature on the Open Dimension Problem concentrates on a very limited number of standard problems, we also refrained from structuring this basic problem type in greater detail at this stage.

Figs. 10 and 11 summarise the system of intermediate problem types which has been introduced here. They can be interpreted as the "landscape" of C & P problems. Also, for each intermediate problem type, an abbreviation derived from the corresponding name is given which allows for a unique identification of each type.

Placement Problem

- one large object ⇒ **Single Large Object Placement Problem**
- several large objects
  - identical large objects ⇒ **(Multiple) Identical Large Object Placement Problem**
  - heterogeneous assortment ⇒ **(Multiple) Heterogeneous Large Object Placement Problem**

Fig. 6: *Intermediate types of the Placement Problem*

Knapsack Problem

- one large object ⇒ **Single Knapsack Problem**
- several large objects
  - identical large objects ⇒ **(Multiple) Identical Knapsack Problem**
  - heterogeneous assortment ⇒ **(Multiple) Heterogeneous Knapsack Problem**

Fig. 7: *Intermediate types of the Knapsack Problem*

Cutting Stock Problem

- identical large objects ⇒ **Single Stock Size Cutting Stock Problem**
- weakly heterogeneous assortment of large objects ⇒ **Multiple Stock Size Cutting Stock Problem**
- strongly heterogeneous assortment of large objects ⇒ **Residual Cutting Stock Problem**

Fig. 8: *Intermediate types of the Cutting Stock Problem*

Bin Packing Type

- identical large objects ⇒ **Single Bin Size Bin Packing Problem**
- weakly heterogeneous assortment of large objects ⇒ **Multiple Bin Size Bin Packing Problem**
- strongly heterogeneous assortment of large objects ⇒ **Residual Bin Packing Problem**

Fig. 9: *Intermediate types of the Bin Packing Problem*

characteristics of the large objects / assortment of the small items		identical	weakly heterogeneous	strongly heterogeneous
all dimensions fixed	one large object	Identical Item Packing Problem IIPP	Single Large Object Placement Problem SLOPP	Single Knapsack Problem SKP
	identical		Multiple Identical Large Object Placement Problem MILOPP	Multiple Identical Knapsack Problem MIKP
	heterogeneous		Multiple Heterogeneous Large Object Placement Problem MHLOPP	Multiple Heterogeneous Knapsack Problem MHKP

Fig. 10: *Landscape of intermediate problem types: output maximisation*

<table>
<tr><td colspan="2">assortment of small items
characteristics of large objects</td><td>weakly heterogeneous</td><td>strongly heterogeneous</td></tr>
<tr><td rowspan="3">all dimensions fixed</td><td>identical</td><td>Single Stock Size Cutting Stock Problem

SSSCSP</td><td>Single Bin Size Bin Packing Problem

SBSBPP</td></tr>
<tr><td>weakly heterogeneous</td><td>Multiple Stock Size Cutting Stock Problem

MSSCSP</td><td>Multiple Bin Size Bin Packing Problem

MBSBPP</td></tr>
<tr><td>strongly heterogeneous</td><td>Residual Cutting StockProblem

RCSP</td><td>Residual Bin PackingProblem

RBPP</td></tr>
<tr><td colspan="2">one large object

variable dimension(s)</td><td colspan="2">Open Dimension Problem

ODP</td></tr>
</table>

Fig. 11: *Landscape of intermediate problem types: input minimisation*

6.3 Refined Problem Types

In a final step, refined problem types are obtained by application of the criterion “dimensionality” and – for two- and three-dimensional problems – of the criterion “shape of the small items”. The resulting subcategories are characterised by adjectives which are added to (the names of) the intermediate problem types (IPT) according to the following system:

{1, 2, 3}-dimensional {rectangular, circular, …, irregular} {IPT} .

7 Specific Problem Categories

In order to make the consequences of the suggested typology clearer, we will now explain the different categories in further detail by giving examples of specific cutting and packing problems (described in the literature) which belong to the problem types introduced above. In particular, well-known standard problems will be pointed out as representatives of the respective categories.

7.1 Identical Item Packing Problem

A well-known (regular / rectangular) representative of this problem type is the ***(Classic) Manufacturer's Pallet Loading (Packing) Problem*** (MPLP; Dowsland 1987, Morabito & Morales 1998), in which a single pallet has to be loaded with a maximal number of identical boxes (the problem itself – without this particular name – has been described even earlier in the literature, see, e.g., Steudel 1979, Smith & deCani 1980). It is usually assumed that the boxes are loaded in layers, in which all boxes have the same vertical orientation. By means of this assumption, the problem is actually reduced to a two-dimensional one (cf. Bischoff & Ratcliff 1995, p. 1322; also see Dyckhoff's (1990, p. 155) characterisation of the problem as being of type 2/B/O/C), namely to the problem of assigning a maximal number of small identical rectangles (representing the "bottom" surfaces of the boxes) to a given large rectangle (representing the pallet). In other words: The MPLP is a two-dimensional, rectangular Identical Item Packing Problem (IIPP). The ***Cylinder Packing Problem*** (Correia, Oliveira & Ferreira 2000, 2001, Birgin, Martinez & Ronconi 2005) possesses an almost identical problem structure, in which – instead of identical rectangles – a maximal number of non-overlapping circles of the same size has to be assigned to a given rectangle (two-dimensional, circular IIPP; also see Isermann 1991). Among others, it can be found in the production of cans where the circles represent the lids and bottoms of (cylindrical) cans to be cut from tin plates, or in logistics management, where cylindrical cans have to be packed on a pallet.

The ***Single-Box-Type Container Packing Problem*** (as discussed in George 1992) is an example of a three-dimensional special case of the Rectangular (i.e. regular) IIPP. It requires the loading of a single container (large object) with a maximum number of identical boxes (small items).

7.2 Single Large Object Placement Problem

The ***Bounded Knapsack Problem*** and the ***Unbounded Knapsack Problem*** (as described in Martello & Toth 1990, pp. 82-91, and pp. 91-103, respectively) are one-dimensional representatives of the SLOPP type. Both require that a (single) knapsack (large object) of given limited weight capacity has to be packed with a selection (subset) from a given set of small items of given weights and values, such that the value of the packed items is maximised. The assortment of the small items is weakly heterogeneous, only relatively few different item types can be identified. The number of times an item of a particular type can be packed may be limited (bounded problem), or unlimited (unbounded problem).

Haims & Freeman (1970) introduce a two-dimensional problem of the SLOPP type which they name ***Template-Layout Problem***. According to their general definition, a weakly heterogeneous set of regular or non-regular small items (“forms”) has to be cut from a single large rectangle such that the value of the cut items is maximised. In the subsequent discussion, the authors concentrate on rectangular small items (i.e. the discussed problem is of the two-dimensional, rectangular SLOPP type). In Christofides & Whitlock 1977, Beasley 1985a, or Christofides & Hadjiconstantinou 1995, e.g., the authors also restrict themselves to rectangular small items. They call it the ***Two-Dimensional Cutting Problem*** (more precisely, we would rather call it the two-dimensional, rectangular SLOPP). The number of times an item of a particular type can be cut from the large rectangle may be constrained explicitly by an upper bound (as in Christofides & Whitlock 1977, Wang 1983, Beasley 1985b, Christofides & Hadjiconstantinou 1995, Beasley 2004). In Scheithauer & Sommerweiß (1998) the unbounded version of this problem (also see Herz 1972; Beasley 1985c) is discussed under the name ***Rectangle Packing Problem***. In the ***(Constrained) Circular Cutting Problem*** (Hifi & M’Hallah 2004; according to the suggested typology, the problem would be characterised as a two-dimensional, circular SLOPP), a rectangular plate (large object) of given size has to be cut down into circular items, which are of m different types (radii). The number of small items which may be cut is bounded, and the objective is to minimise the unused space of the plate.

What has been called the ***Single Container Loading (Packing) Problem*** in the literature (George & Robinson 1980, Bortfeldt, Gehring & Mack 2003) is an example of the three-dimensional, rectangular SLOPP. It requires loading a fairly large, weakly heterogeneous consignment of boxes into a given container such that the volume or value of the packed boxes is maximised, or, equivalently, the unused space of the container or the value of the unpacked boxes is minimised (also see Ratcliff & Bischoff 1998, Bortfeldt & Gehring 1998). A set of weakly heterogeneous cargo has to be packed on a pallet in the ***Distributor’s (Single) Pallet Loading Problem*** (Hodgson 1982, Bischoff, Janetz & Ratcliff 1995). Unlike the Manufacturer’s Pallet Loading Problem, this is a truly three-dimensional rectangular problem, as – due to the different box sizes – it may not be sufficient to concentrate on layered packing if the space available above the pallet is to be used in the best possible way. In real-world Single Container and Pallet Loading Problems the number of boxes which are to be packed of a particular box type will be limited by an upper bound. Hifi (2004), on the other hand, considers the unconstrained case.

7.3 Multiple Identical Large Object Placement Problem

Straightforward extensions of the Single Large Object Placement Problem could take into account multiple identical large objects. This could give rise, e.g., to the (Bounded) Multiple Knapsack Problem, the Multiple Stock Sheet Cutting Problem etc. Problems of this type (MILOPP), however, have not yet been discussed in the literature, at least not to the knowledge of the authors of this paper.

7.4 Multiple Heterogeneous Large Object Placement Problem

Another extension of the Single Large Object Placement Problem which takes into account multiple, non-identical large objects also appears to having been treated very rarely in the literature. A one-dimensional cutting problem is described in Gradišar et al. (1999, p. 559, case 2), in which a weakly heterogeneous assortment of order lengths has to be cut from a set of input lengths which are all different (one-dimensional MHLOPP). The supply of input lengths is not sufficient to satisfy all orders. Therefore, the objective is to minimise the total size of the uncut order lengths.

Eley (2003) considers three-dimensional container loading problems, in which a weakly heterogeneous consignment of boxes has to be packed into containers available in different sizes. In one of the discussed situations, the available container space is not sufficient to accommodate all boxes. Therefore, a selection of boxes has to be determined that maximises the volume utilisation, or, alternatively, the value of the packed boxes (three-dimensional, rectangular MHLOPP) . An exact algorithm for the unconstrained version of this problem is introduced in Hifi (2004).

7.5 Single Knapsack Problem

The ***Classic (One-Dimensional) Knapsack Problem***, also called ***0-1-Knapsack Problem*** (Martello, Pisinger & Toth 2000, Martello & Toth 1990, p. 13) which requires packing a given set of (different) items of given weights and values into a (single) knapsack of given limited weight capacity such that the value of the packed items is maximised (cf. Martello & Toth 1990, pp. 3, 13-77) obviously is a special case of this problem type (i.e. the one-dimensional SKP). The ***Subset-Sum Problem*** (cf. Martello & Toth 1990, pp. 105ff.) is a 0-1-Knapsack Problem in which the weight of an item is identical with its value. A straightforward extension of this problem, namely the ***Multiconstraint (Zero-One) Knapsack Problem*** (Gavish & Pirkul 1985, Drexl 1988, Thiel & Voss 1994, the problem is also called ***m-Constraint Knapsack Problem***, cf. Schilling 1990), also belongs to this category. Apart from the capacity constraint, m-1 additional constraints have to be satisfied by the packed items.

Extensions of the Classic (One-Dimensional) Knapsack Problem into two and more geometric dimensions give rise to the ***Two-Dimensional (Single Orthogonal) Knapsack Problem*** (Caprara & Monaci 2004; also see e.g. Healy & Moll 1996), in which a set of small, distinct rectangles has to be cut from a single large rectangle (two-dimensional, rectangular SKP), and the ***Three-Dimensional (Single Orthogonal) Knapsack Problem*** (also called ***Knapsack Container Loading Problem***; cf. Pisinger 2002), in which rectangular-shaped boxes have to be packed into a container (three-dimensional, rectangular SKP; also see Bischoff & Marriott 1990, Scheithauer 1999, Bortfeldt & Gehring 2001). In both cases usually the value of the cut / packed small items is to be maximised. If their value can be assumed to be proportional to their size / volume, then equivalently the unused space of the large rectangle or the container can be minimised. Fekete & Schepers (1997) finally looks at the ***n-Dimensional (Single Orthogonal) Knapsack Problem***, which extends the Classic Knapsack Problem into n geometric dimensions.

George, George & Lamar (1995) describe a two-dimensional, circular SKP, in which (the bottom of) a container (rectangle) has to be filled with a set of distinct pipes (circles) such that the value of the selected pipes is maximised.

7.6 Multiple Identical Knapsack Problem

In the literature, a specific one-dimensional case of the Multiple Identical Knapsack Problem is known as the ***Maximum Cardinality Bin Packing Problem***, in which a fixed number of large objects with a given, identical capacity and a (strongly heterogeneous) set of small, indivisible items of given weights are given. The objective is to maximise the number of packed items (Labbé, Laporte & Martello 2003).

The ***Multiple Container Packing Problem*** (as described in Raidl & Kodydek 2003) is a three-dimensional (regular) special case of this problem category, in which a given number of identical containers has to be filled with a (strongly heterogeneous) set of items of given weights and values. The total value of the packed items has to be maximised (three-dimensional, rectangular SKP). According to the authors' knowledge, this is the only representative of this problem type discussed in the literature so far.

7.7 Multiple Heterogeneous Knapsack Problem

Martello & Toth (1990, pp. 157-187, also see Pisinger 1999) consider the ***0-1 Multiple Knapsack Problem***, which is a one-dimensional representative of this problem type (i.e. a one-dimensional MHKP). A strongly heterogeneous set of small items, each of which is characterised by a specific weight and profit, has to be packed into a set of knapsacks of distinct capacities. For each knapsack, the packed items must not exceed the available capacity, and the total profit of the small items which are packed has to be maximised.

7.8 Open Dimension Problem

In the literature, Open Dimension Problems are usually discussed for a single large object. Obviously, problems of this kind are only possible in two and more dimensions.

The ***(Two-Dimensional) Strip Packing Problem*** is an Open Dimension Problem in which a set of two-dimensional small items has to be laid out on a rectangular large object; the width of the large object is fixed, its length is variable and has to be minimised. In case the small items are rectangles (cf. Kröger 1995, Jacobs 1996, Hopper & Turton 2001a, Martello, Monaci & Vigo 2003) one may also refer to this problem as the ***Rectangular Strip Packing Problem***, or even as the ***Orthogonal Rectangular Strip Packing Problem***, if the small rectangles have to be laid out on the large object orthogonally. If the rectangles have to be packed "in levels", this problem is also called ***Level Packing Problem*** (cf. Lodi, Martello & Vigo 2004).

In case the small items are non-regularly shaped objects (Oliveira & Ferreira 1993, Bennell & Dowsland 2001), like in the shoe-manufacturing industry, where they may represent pieces of shoes to be cut from a roll of leather, the problem (i.e. in case of the two-dimensional, irregular ODP, according to the notation introduced here) is also referred to as the ***Irregular Strip Packing Problem*** (Hopper & Turton 2001b, p. 257) or the ***Nesting Problem*** (cf. Oliveira & Ferreira 1993, Oliveira, Gomes & Ferreira

2000, Carravilla, Ribeiro & Oliveira 2003). With respect to applications in specific areas, the same problem may also be known under different names (e.g. as the ***Marker-Making Problem*** in the apparel industry; cf. Li & Milenkovic 1995).

A three-dimensional, rectangular Open Dimension Problem with a single variable dimension (length) occurs in distribution planning, when a given set of cargo (small items) has to be fitted into a container in such a way that the least space in terms of container length is used (Scheithauer 1991, Miyazawa & Wakabayashi 1997).

In the ***Minimal Enclosure Problem*** (Milenkovic & Daniels 1999), also called ***Rectangular Packing Problem*** (Hifi & Ouafi 1998), a set of two-dimensional items has to be laid out such that it can be included in a rectangular (large) object of minimal area. In this case, the extension of the large objects in both dimensions (width and length) is variable. The small items may be of rectangular (cf. Hifi & Ouafi 1998), circular (Stoyan & Yaskov 1998), or irregular shape (Milenkovic & Daniels 1999).

At least to our knowledge, Open Dimension Problems with more than one large object have not been discussed frequently in the area of C & P. Benati (1997) discusses a two-dimensional problem with several large objects of different widths which all have an infinite length. However, we would like to point out that the well-studied **(*Classic*) *Multiprocessor Scheduling Problem*** could be interpreted as a one-dimensional problem of this kind. In this problem, a given set of indivisible jobs (small items) with given processing times (length) has to be allocated to a given number of identical processors (large objects) such that the maximal completion time (also called “schedule length”) is minimised (cf. Heuer 2004; Brucker 2004, pp. 107-154; Błażewicz et al. 2001, pp. 137-203). The completion time of a processor is the time necessary to process all the jobs which have been assigned to this processor. In other words, an identical, minimal time-capacity (extension in the variable dimension) has to be assigned to each of the processors that is large enough that all the jobs can be completed. Due to the fact that the Multiprocessor Scheduling Problem and other related problems are hardly ever discussed with respect to C & P problems, we exclude Open Dimension Problems with more than one large object from the following considerations.

7.9 Single Stock-Size Cutting Stock Problem

Problems of this type include the ***Classic One-Dimensional Cutting Stock Problem*** (Gilmore & Gomory 1963, Wäscher & Gau 1996), in which standard (or: stock) material of a specific, single length has to be cut down into a weakly heterogeneous set of order lengths (small items). In the ***Classic Two-Dimensional Cutting Stock Problem*** (Gilmore & Gomory 1965), a weakly heterogeneous set of order rectangles has to be cut from stock plates of a specific, single size (length and width). In both problems, the number or the value of the necessary large objects (stock lengths or stock plates) has to be minimised.

Examples for a three-dimensional (rectangular) problem of this type include the ***Multi-Pallet Loading Problem*** (as discussed in Terno et al. 2000) and the ***Multi-Container Loading Problem*** (Scheithauer 1999), in which a weakly heterogeneous assortment of cargo (i.e. a set of boxes) is to be packed on a minimum number of pallets or into a minimum number of containers (also see Bortfeldt 2000).

7.10 Multiple Stock-Size Cutting Stock Problem

Problems of this kind include the natural extensions of the One-Dimensional and Two-Dimensional Cutting Stock Problems to more than one stock size (see Gilmore & Gomory 1961, Rao 1976, Dyckhoff 1981, Scheithauer 1991, Belov & Scheithauer 2002 for the one-dimensional and Riehme, Scheithauer & Terno 1996, Morabito & Arenales 1996 for the two-dimensional, rectangular case). The one-dimensional Multiple Stock-Size Cutting Stock Problem has also been considered under the name “Paper Trim Problem” in the literature (Golden 1976, p. 265f.). For a two-dimensional application (i.e. a two-dimensional, rectangular MSSCSP) from furniture manufacturing see Carnieri, Mendoza & Gavinho 1994.

One of the container-loading problems discussed in Eley 2003 represents a three-dimensional (rectangular) case of this problem type. Both containers and boxes can be grouped into classes. Associated with each container type are specific costs, the total costs of the containers necessary to accommodate all boxes are to be minimised.

7.11 Residual Cutting Stock Problem

For a Cutting Stock Problem with a strongly heterogeneous assortment of large objects we have chosen the name “Residual Cutting Stock Problem” here, because in practice this case comes about whenever large objects are to be used which represent unused parts of input material (“left-overs”) from previous C & P processes. Under the name of “***Hybrid One-Dimensional Cutting Stock Problem***” a one-dimensional case of this problem type has been introduced in the literature (cf. Gradišar, Resinovič & Kljajić 2002, p. 1212, Gradišar, Kljajić & Resinovič 1999). The authors introduce a one-dimensional cutting problem, in which a weakly heterogeneous assortment of order lengths has to be cut from a set of input lengths which are all different. The supply of input lengths is sufficient to satisfy the demands, and the objective is to minimise the trim loss of the input lengths which are to be used (also see Gradišar et al. 1999, p. 559, case 1). They argue that traditional, pure item-oriented or pure pattern-oriented solution approaches are not appropriate. Instead, they suggest a new solution approach which is a combination of both. The two-dimensional case is considered in Vanderbeck 2001. Extensions of this problem type into three or even more dimensions have not been discussed in the literature so far.

7.12 Single Bin-Size Bin Packing Problem

The ***Classic (One-Dimensional) Bin Packing Problem*** is a representative of this problem type. It requires packing a given set of distinct small items of given weights to a minimal number of large objects (bins) of identical size (capacity) such that for each bin the total capacity of the small items does not exceed its capacity (cf. Martello & Toth 1990, Scholl, Klein & Jürgens 1997, Schwerin & Wäscher 1997). We also note that this problem type has also been named ***Vehicle Loading Problem*** (cf. Golden 1976, p. 266) and ***Binary Cutting Stock Problem*** (cf. Vance et al. 1994) in the literature. The ***k-item Bin Packing Problem*** (cf. Babel et al. 2004) additional requires the assignment of at most k items to each bin.

What is called the ***Two-Dimensional (Orthogonal) Bin Packing Problem*** (Lodi, Martello & Vigo 1999, 2002a, p. 379, Lodi, Martello & Monaci 2002, p. 242; Martello & Vigo 1998 also refer to this problem type as the ***Two-Dimensional Finite Bin Packing Problem*** in order to distinguish it from the Two-Dimensional Strip Packing Problem, in which the large object has an infinite extension in one dimension) consists of assigning a set of distinct rectangles orthogonally to a minimum number of rectangular bins. According to the suggested typology this is a two-dimensional, rectangular SBSBPP type. George, George & Lamar (1995, p. 693) mention the ***Cylindrical Bin Packing Problem***. This is a two-dimensional circular SBSBPP, in which the large objects are rectangles and the small items are circles. Real problems of this kind arise in logistics when (a minimum number of) containers are to be loaded with pipes.

In the ***Three-Dimensional (Orthogonal) Bin Packing Problem*** the items are assumed to be rectangular boxes which are to be fitted orthogonally into a minimal number of rectangular containers of identical size (cf. Lodi, Martello & Vigo 2002b). The ***Cube Packing Problem*** is a special case of the three-dimensional rectangular Bin Packing Problem, in which all boxes and bins are cubes (Miyazawa & Wakabayashi 2003).

7.13 Multiple Bin-Size Bin Packing Problem

The ***(One-Dimensional) Variable-Sized Bin Packing Problem*** (Chu & La 2001, Kos & Duhovnik 2002) is an extension of the Classic One-Dimensional Bin Packing Problem in which several bin types are introduced (i.e. it is a one-dimensional MBSBPP). Each bin type is in unlimited supply and characterised by specific costs and size. All the small items have to be assigned to bins, and the total costs of the used bins have to be minimised (cf. Kang & Park 2003). A two-dimensional, rectangular case is considered in Tarasova, Razanova & Gabitov (1997), in which the large objects are in limited supply and where the total area of the material necessary to cut all small items is to be minimised.

7.14 Residual Bin Packing Problem

In analogy to the Cutting Stock Problem, for this problem type which is characterised by a set of strongly heterogeneous large objects we have chosen the name "***Residual Bin Packing Problem"***.

Chen, Lee & Shen (1995) consider a three-dimensional container loading problem, in which a weakly heterogeneous consignment of boxes has to be packed into containers available in different sizes. In one of the discussed situations, the available container space is not sufficient to accommodate all boxes. Therefore, a selection of boxes has to be determined that maximises the volume utilisation, or, alternatively, the value of the packed boxes (three-dimensional, rectangular RBPP).

8 Categorisation of Recent Literature (1995-2004)

In order to demonstrate the practicability and usefulness of the suggested typology, the recent C & P literature has been reviewed and categorised according to the scheme introduced above. We concentrated on papers which are publicly available and have been published in English in international journals, edited volumes, or conference proceedings during the decade between 1995 and 2004. Monographs and working papers have not been considered in our investigation.

In order to exclude publications from the periphery of C & P from our analysis, which are only of marginal interest to researchers and practitioners in the field, we restricted ourselves to papers related to C & P problems “in a narrow sense” (Dyckhoff 1990, p. 148) in the first place. Papers on “abstract” C & P problems (Dyckhoff 1990, p. 148) have only been considered as far as these problems (like those of the knapsack type) have been introduced in the suggested typology, i. e. we refrained from including other papers from this category not addressing C & P directly, like papers on flow-line balancing (Talbot, Patterson & Gehrlein 1986, Scholl 1995), multi-processor scheduling (Brucker 2001, pp. 107-154, Błażewicz et. al. 2001, pp. 137-203), or capital budgeting (Lorie & Savage 1955).

Furthermore, only papers dealing with C & P problems in the sense of refined problem types have been included in our analysis. Articles dealing with problem extensions and problem variants were not taken into account for. We note again that we have excluded papers of this kind only in order to keep the number of papers to be considered to a manageable size and to find a definition of a paper cluster that is of interest to researchers and practitioners in the field of C & P. Focussing our investigation in this particular way by no means limits the usefulness and the value of the suggested typology. In fact, also the problems discussed in papers which have been excluded here can be categorised according to our typology.

Finally we remark that only such papers have been categorised which strictly satisfy the above-given definition of C & P problems and subsequent specifications. That means that papers e.g. dealing with divisable small items and/or large objects (Kang & Park 2003), or allowing the “overpacking” of bins (Coffman & Lueker 2001) do not appear in our analysis.

As far as for December 2005, 413 papers have been identified containing material relevant in the above-described sense. These papers are listed at http://www.uni-magdeburg.de/mansci/rm/cp_typology, together with the corresponding problem types to which they have been assigned. Table 1 shows what problem types and number of dimensions are dealt with in these papers. The total numbers given in this and the following tables is larger than 413 because in some papers more than one problem type is addressed. Consequently, those papers had to be counted more than once.

Research on input minimisation problems (dealt with in 263 papers, 59 percent) clearly dominates research on output maximisation problems (182 papers, 41 percent). One-dimensional and two-dimensional problems (172 papers, 39 percent, and 214 papers, 48 percent) are considered significantly more often than three-dimensional ones (59 papers, 13 percent).

kind of assignment	1D	2D regular	2D irregular	3D	total
input minimisation	108	79	52	24	263
output maximisation	64	71	12	35	182
total	172	150	64	59	445

Table 1: *Number of problem-relevant dimensions and assignment types of problems dealt with in the literature*

Fig. 12 and 13 give a more detailed analysis of how the papers are distributed over the different problem categories. Among research on output maximisation problems, papers on problems with a single large object are prevailing (165 papers, 37 percent of all papers and 91 percent of those belonging to the output-maximisation type). With respect to input minimisation, most of the publications deal with problems in which the assortment of large objects either consists of identical large objects of a single given size (127 papers, 29 percent of all papers, and 48 percent of those belonging to the input-minimisation type), or, alternatively, of a single large object with one variable dimension (102 papers, 23 and 39 percent).

assortment of the small items / characteristics of the large objects		identical	weakly heterogeneous	strongly heterogeneous
all dimensions fixed	one large object	IIPP 23	SLOPP 56	SKP 86
	identical		MILOPP 1	MIKP 6
	heterogeneous		MHLOPP 4	MHKP 6

Fig. 12: *Distribution of publications: output maximisation*

<table>
<tr><td colspan="2">assortment of small items
characteristics of large objects</td><td>weakly heterogeneous</td><td>strongly heterogeneous</td></tr>
<tr><td rowspan="3">all dimensions fixed</td><td>identical</td><td>SSSCSP
38</td><td>SBSBPP
89</td></tr>
<tr><td>weakly heterogeneous</td><td>MSSCSP
18</td><td>MBSBPP
4</td></tr>
<tr><td>strongly heterogeneous</td><td>RCSP
10</td><td>RBPP
2</td></tr>
<tr><td colspan="2">one large object

variable dimension(s)</td><td colspan="2">ODP
102</td></tr>
</table>

Fig. 13: *Distribution of publications: input minimisation*

As becomes further evident, published research concentrates on five problem types, namely on the ODP (102 papers, 23 percent), SBSBPP (89 papers, 20 percent), SKP (86 papers, 19 percent), SLOPP (56 papers, 13 percent) and the SSSCSP (38 papers, 9 percent). Papers on these five problem types account for 371 out of 445 publications (83 percent). In Table 2 the numbers of papers which consider these problem types are further differentiated with respect to the number of problem-relevant dimensions.

problem types	1D	2D regular	2D irregular	3D	total
ODP	-	46	49	7	102
SBSBPP	61	17	2	9	89
SKP	49	18	7	12	86
SLOPP	4	32	1	19	56
SSSCSP	29	2	1	6	38
other	29	35	4	6	74
total	172	150	64	59	445

Table 2: *Number of papers on selected problem types, differentiated according to the number of problem-relevant dimensions*

Table 2 reveals that research on C & P still is rather traditionally oriented. It stresses areas which include clearly-defined ("classic") standard problems, well-studied for three decades or an even longer period of time, such as the one-dimensional SBSBPP (including the Classic Bin Packing Problem), the two-dimensional ODP (including the Regular and the Irregular Strip-Packing Problem), the one-dimensional SSSCSP (including the Classic Cutting Stock Problem), the one-dimensional SKP (including the Classic Knapsack Problem), and the two-dimensional SLOPP (including the Distributor's Pallet Loading Problem). As far as research departs from these traditional areas, it is devoted to straightforward extensions of these standard problems into a higher number of dimensions, e.g. to the two-dimensional Bin Packing Problem, the two-dimensional Knapsack Problem etc. Other kinds of problem extensions (e.g. considering different assortments of large objects and small items) which are probably more relevant to the solution of real-world C & P problems can only be found far less commonly in the literature, even though a few, more recent papers considering a heterogeneous assortment of large objects (51) seem to indicate that the preferences of researchers might be changing.

We finally note (cf. Fig. 12) that the IIPP which has been extensively studied in the 1980's is only represented in 23 papers. This seems to indicate that the central standard problem of this type, the Classic Manufacturer's Pallet Loading Problem, has been solved satisfactorily and that research has shifted to more complex problem situations recently, taking care, in particular, of a (weakly) heterogeneous assortment of small items (boxes to be packed).

9 Outlook

The suggested typology should be sufficient for an initial, brief orientation in a particular area of C & P. To practitioners and researchers confronted with particular C & P problems Fig. 2 points out the relevant problem parameters, allowing them to assign their problems to the relevant problem categories. An extensive database of C & P publications (also including those concerning problem extensions and problem variants), in which papers are classified and organised in accordance with the suggested typology, is available at the ESICUP webpage (http://www.apdio.pt/sicup/). It provides direct access to the literature of each specific problem type.

Fig. 2 also outlines the area of future work. In order to structure the refined problem types further, it will be necessary to compile, analyse and group the respective additional constraints. Furthermore, problem extensions and problem variants and the corresponding standard problems have to be investigated in greater detail.

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Literature

Abdou, G.; Elmasry, M. (1999):
3D random stacking of weakly heterogeneous palletization problems. *International Journal of Production Research* **37**, 1505-1524.

Babel, L., Chen, B., Kellerer, H., Kotov, V. (2004):
Algorithms for on-line bin-packing problems with cardinality constraints. *Discrete Applied Mathematics* **143**, 238-251.

Beasley, J. E. (1985b):
An exact two-dimensional non-guillotine cutting tree search procedure. *Operations Research* **33**, 49-64.

Beasley, J. E. (1985c):
Algorithms for unconstrained two-dimensional guillotine cutting. *Journal of the Operational Research Society* **36**, 297-306.

Beasley, J. E. (2004):
A population heuristic for constrained two-dimensional non-guillotine cutting. *European Journal of Operational Research* **156**, 601-627.

Belov, G.; Scheithauer, G. (2002):
A cutting plane algorithm for the one-dimensional cutting stock problem with multiple stock lengths. *European Journal of Operational Research* **141**, 274-294.

Benati, S. (1997):
An algorithm for a cutting stock problem on a strip. *Journal of the Operational Research Society* **48**, 288-294.

Bennell, J. A.; Dowsland, K. A. (2001):
Hybridising tabu search with optimisation techniques for irregular stock cutting. *Management Science* **47**, 1160-1172.

Birgin, E. G.; Martinez, J. M.; Ronconi, D. P. (2005):
Optimizing the packing of zylinders into a rectangular container: A nonlinear approach. *European Journal of Operational Research* **160**, 19-33.

Bischoff, E. E.; Janetz, F.; Ratcliff, M. S. W. (1995):
Loading pallets with non-identical items. *European Journal of Operational Research* **84**, 681-692.

Bischoff, E. E.; Marriott, M. D. (1990):
A comparative evaluation of heuristics for container loading. *European Journal of Operational Research* **44**, 267-276.

Bischoff, E. E.; Ratcliff, M. S. W. (1995):
Loading Multiple Pallets. *Journal of Operational Research Society* **46**, 1322-1336.

Błażewicz, J.; Ecker, K. H.; Pesch, E.; Schmidt, G. Węglarz, J. (2001):
Scheduling computer and manufacturing processes. Springer, Berlin et al.

Bortfeldt, A. (2000):
Eine Heuristik für Multiple Containerladeprobleme. *OR Spektrum* **22**, 239-261.

Bortfeldt, A.; Gehring, H. (1998):
Ein Tabu-Search-Verfahren für Containerbeladeprobleme mit schwach heterogenen Kistenvorrat. *OR Spektrum* **20**, 237-250.

Bortfeldt, A.; Gehring, H. (2001):
A hybrid genetic algorithm for the container loading problem. *European Journal of Operational Research* **131**, 143-161.

Bortfeldt, A.; Gehring, H.; Mack, D. (2003):
A parallel tabu search algorithm for solving the container loading problem. *Parallel Computing* **29**, 641-662.

Brucker, P. (2004):
Scheduling algorithms. 4th edition, Springer, Berlin et al.

Caprara, A.; Monaci, M. (2004):
On the two-dimensional Knapsack Problem. *Operations Research Letters* **32**, 5-14.

Carnieri, C., Mendoza, G. A., Gavinho, L. G. (1994):
Solution procedures for cutting lumber into furniture parts. *European Journal of Operational Research* **73**, 485-501.

Carravilla, M. A.; Ribeiro, C. ; Oliveira, J. F. (2003):
Solving nesting problems with non-convex polygons by constraint logic programming. *International Transactions in Operational Research* **10**, 651-664.

Chen, C. S.; Lee, S. M.; Shen, Q. S. (1995):
An analytical model for the container loading problem. *European Journal of Operational Research* **80**, 68-76.

Christofides, N.; Whitlock, C. (1977):
An algorithm for two-dimensional cutting problems. *Operations Research* **25**, 30-44.

Christofides, N.; Hadjiconstantinou, E. (1995):
An exact algorithm for general, orthogonal, two-dimensional knapsack problems. *European Journal of Operational Research* **83**, 39-56.

Chu, C.; La, R. (2001): Variable-sized bin packing:
Tight absolute worst-case performance ratios for four approximation algorithms. *SIAM Journal on Computing* **30**. 2069-2083.

Coffman, E. G.; Lueker, G. S.(2001):
Approximation algorithms for extensible bin packing. In: Proceedings of the 12th Annual ACM-SIAM Symposium on Discrete Algorithms. Society for Industrial and Applied Mathematics, Philadelphia, 586-588.

Correia, M. H.; Oliveira, J. F.; Ferreira, J. S. (2000):
Cylinder packing by simulated annealing. *Pesquisa Operacional* **20**, 269-286.

Correia, M. H.; Oliveira, J. F.; Ferreira, J. S. (2001):
A new upper bound for the cylinder packing problem. *International Transactions in Operatinal Research* **8**, 571-583.

Das, S.; Ghosh, D. (2003):
Binary knapsack problems with random budgets. *Journal of the Operational Research Society* **54**, 970-983.

Dowsland, K. A. (1987):
An exact algorithm for the pallet loading problem. *European Journal of Operational Research* **31**, 78-84.

Drexl, A. (1988):
A simulated annealing approach to the multiconstraint zero-one knapsack problem. *Computing* **40**,1-8.

Dyckhoff, H. (1981):
A new linear programming approach to the cutting stock problem. *Operations Research* **6**, 1092-1104.

Dyckhoff, H. (1990):
A typology of cutting and packing problems. *European Journal of Operational Research* **44**, 145-159.

Eley, M. (2003):
A bottleneck assignment approach to the multiple container loading problem. *OR Spectrum* **25**, 45-60.

Faroe, O.; Pisinger, D.; Zachariasen, M. (2003):
Guided Local Search for the Three-Dimensional Bin-Packing Problem. *INFORMS Journal on Computing* **15**, 267-283.

Fekete, S. P., Schepers, J. (1997): A new exact algorithm for general orthogonal d-dimensional knapsack problems. In: Burkhard, R., Woeginger, G. J. (Eds.): Proceedings of the 5th Annual European Symposium on Algorithms, Lecture Notes in Computer Science 1284, Springer, Berlin et al., 144-156.

Foerster, H.; Wäscher, G. (1998):
Simulated annealing for order spread minimization in sequencing cutting patterns. *European Journal of Operational Research* **110**, 272-281.

Gavish, B.; Pirkul, H. (1985):
Efficient algorithms for solving multiconstraint zero-one knapsack problems to optimality. *Mathematical Programming* **31**, 78-105.

George, J. A.; Robinson, D. F. (1980):
A heuristic for packing boxes into a container. *Computers & Operations Research* **7**, 147-156.

George J.A. (1992):
A method for solving container packing for a single size of box. *Journal of the Operations Research Society* **43**, 307-312.

George, J. A.; George, J. M.; Lamar, B. W. (1995):
Packing different-sized circles into a rectangular container. *European Journal of Operational Research* **84**, 693-712.

Gilmore, P. C.; Gomory, R. E. (1961):
A linear programming approach to the cutting-stock problem part I. *Operations Research* **9**, 849-859.

Gilmore, P. C.; Gomory, R. E. (1963):
A linear programming approach to the cutting-stock problem part II. *Operations Research* **11**, 864-888.

Gilmore, P. C.; Gomory, R. E. (1965):
Multistage cutting stock problems of two and more dimensions. *Operations Research* **13**, 94-120.

Golden, B. L. (1976):
Approaches to the cutting stock problem. *AIIE Transactions* **8**, 265-274.

Gradišar, M.; Resinovič, G.; Kljajić, M. (1999):
A hybrid approach for optimization of one-dimensional cutting. *European Journal of Operational Research* **119**, 719-728.

Gradišar, M.; Kljajić, M.; Resinovič, G.; Jesenko, J. (1999):
A sequential heuristic procedure for one-dimensional cutting. *European Journal of Operational Research* **114**, 557-568.

Gradišar, M.; Resinovič, G.; Kljajić, M. (2002):
Evaluation of algorithms for one-dimensional cutting. *Computers & Operations Research* **29**, 1207-1220.

Haims, M. J.; Freeman, H. (1970):
A multistage solution of the template-layout problem. *IEEE Transactions on Systems Science and Cybernetics* **SSC-6**, 145-151.

Healy, P.; Moll, R. (1996):
A Local Optimization-based Solution to the Rectangle Layout Problem. *Journal of the Operational Research Society* **47**, 523-537.

Hemminki, J.; Leipälä, T.; Nevalainen, O. (1998):
On-line packing with boxes of different sizes. *International Journal of Production Research* **36**, 2225-2245.

Herz, J. C. (1972):
Recursive computational Procedure for two-dimensional stock cutting. *IBM Journal of Research Development* **16**, 462-469.

Heuer, J. (2004):
Das Multiprocessor Scheduling-Problem mit reihenfolgeabhängigen Rüstzeiten. Deutscher Universitäts-Verlag, Wiesbaden.

Hifi, M.; Ouafi, R. (1998):
A best-first branch-and-bound algorithm for orthogonal rectangula packing problems. *International Transactions in Operational Research* **5**, 345-356.

Hifi, M. (2004):
Exact algorithms for unconstrained three-dimensional cutting problems: a comparative study. *Computers & Operations Research* **31**, 657-674.

Hifi, M.; M'Hallah, R. (2004):
Approximate algorithms for constrained circular cutting problems. *Computers & Operations Research* **31**, 675-694.

Hodgson, T. J. (1982):
A Combined Approach to the Pallet Loading Problem. *IIE Transactions* **14**, 175-182.

Hopper, E.; Turton, B. C. H. (2001a):
An empirical investigation of meta-heuristic and heuristic algorithms for a 2D packing problem. *European Journal of Operational Research* **128**, 34-57.

Hopper, E.; Turton, B. C. H. (2001b):
A review of the application of meta-heuristic algorithms to 2D strip packing problems. *Artificial Intelligence Review* **16**, 257-300.

Isermann, H. (1991).
Heuristiken zur Lösung des zweidimensionalen Packproblems für Rundgefäße. *OR Spektrum* **13**, 213-223.

Jacobs, S. (1996):
On generic algorithms for the packing of polygons. *European Journal of Operational Research* **88**, 165-181.

Kang, J.; Park, S. (2003):
Algorithms for the variable sized bin packing problem. *European Journal of Operational Research* **147**, 365-372.

Katagiri, H.; Ishii, H.; Sakawa, M. (2004):
On fuzzy random linear knapsack problems. *Central European Journal of Operations Research* **12**, 59-70.

Kos, L.; Duhovnik, J. (2002):
Cutting optimization with variable-sized stock and inventory status data. *International Journal of Production Research* **40**, 2289-2301.

Kröger, B. (1995):
Guillotineable bin packing: A genetic approach. *European Journal of Operational Research* **84**, 645-661.

Labbé, M.; Laporte, G.; Martello, S. (2003):
Upper bounds and algorithms for the maximum cardinality bin packing problem. *European Journal of Operational Research* **149**, 490-498.

Li, Z.; Milenkovic, V. (1995):
Compaction and separation algorithms for non-convex polygons and their applications. *European Journal of Operational Research* **84**, 539-561.

Lins, L.; Lins, S.; Morabito, R. (2002):
An n-tet graph approach for nonguillotine packings of n-dimensional boxes into an n-container. *European Journal of Operational Research* **141**, 421-439.

Lodi, A.; Martello, S.; Vigo, D. (1999):
Approximation algorithms for the oriented two-dimensional bin packing problem. *European Journal of Operational Research* **112**, 158-166.

Lodi, A.; Martello, S.; Vigo, D. (2002a):
Recent advances on two-dimensional bin packing problems. *Discrete Applied Mathematics* **123**, 379-396.

Lodi, A.; Martello, S.; Vigo, D. (2002b):
Heuristic algorithms for the three-dimensional bin packing problem. *European Journal of Operational Research* **141**, 410-420.

Lodi, A.; Martello, S.; Vigo, D. (2004):
Models and Bounds for Two-Dimensional Level Packing Problems. *Journal of Combinatorial Optimization* **8**, 363-380.

Lodi, A.; Martello, S.; Monaci, M. (2002):
Two-dimensional packing problems: A survey. *European Journal of Operational Research* **141**, 241-252.

Lorie, J. H.; Savage, L. J. (1955):
Three Problems in Rationing Capital. *Journal of Business* **28**. 229-239.

Martello, S.; Toth, P. (1990):
Knapsack problems – Algorithms and computer implementations. John Wiley & Sons, Chichester et al.

Martello, S.; Pisinger, D.; Toth, P. (2000):
New trends in exact algorithms for the 0-1 knapsack problem. *European Journal of Operational Research* **123**, 325-332.

Martello, S.; Monaci, M.; Vigo, D. (2003):
An exact approach to the strip-packing problem. *INFORMS Journal on Computing* **15**, 310-319.

Martello, S.; Vigo, D. (1998):
Exact solution of the two-dimensional finite bin packing problem. *Management Science* **44**, 388-399.

Milenkovic, V. J.; Daniels, K. (1999):
Translational polygon containment and minimal enclosure using mathematical programming. *International Transactions in Operational Research* **6**, 525-554.

Miyazawa, F. K.; Wakabayashi, Y. (1997):
An algorithm for the three-dimensional packing problem with asymptotic performance analysis. *Algorithmica* **18**, 122-144.

Miyazawa, F. K.; Wakabayashi, Y. (2003):
Cube Packing. *Theoretical Computer Science* **297**, 355-366.

Morabito, R.; Arenales, M. N. (1996):
Staged and constrained two-dimensional guillotine cutting problems: An AND/OR-graph approach. *European Journal of Operational Research* **94**, 548-560.

Morabito, R.; Morales, S. (1998):
A simple and effective recursive procedure for the manufacturer's pallet loading problem. *Journal of the Operational Research Society* **49**, 819-828.

Nonås, S. L.; Thorstenson, A. (2000):
A combined cutting-stock and lot-sizing problem. *European Journal of Operational Research* **120**, 327-342.

Oliveira, J. F.; Ferriera, J. S. (1993):
Algorithms for nesting problems. In: R. V. V. Vidal (Ed.): Applied Simulated Annealing. Lecture Notes in Economics and Mathematical Systems 396, Springer, Berlin et al., 255-274.

Oliveira, J. F.; Gomes, A. M.; Ferreira, J. S. (2000):
TOPOS - A new constructive algorithm for nesting problems. *OR Spektrum* **22**, 263-284.

Pisinger, D. (1999):
An exact algorithm for large multiple knapsack problems. *European Journal of Operational Research* **114**, 528-541.

Pisinger, D. (2002):
Heuristics for the container loading problem. *European Journal of Operational Research* **141**, 382-392.

Raidl, G. R.; Kodydek, G. (1998):
Genetic Algorithms for the Multiple Container Packing Problem. In: Eiben, A. E., Bäck, T., Schoenauer, M., Schwefel, H.-P. (Eds.): Parallel Problem Solving from Nature. Lecture Notes in Computer Science 1498, Springer, Berlin et al., 875-884.

Rao, M. R. (1976):
On the cutting stock problem. *Journal of the Computer Society of India* **7**, 35-39.

Ratcliff, M. S. W.; Bischoff, E. E. (1998):
Allowing for weight considerations in container loading. *OR Spektrum* **20**, 65-71.

Riehme, J.; Scheithauer, G.; Terno, J. (1996):
The solution of two-stage guillotine cutting stock problems having extremely varying order demands. *European Journal of Operational Research* **91**, 543-552.

Scheithauer, G. (1991):
A three-dimensional bin packing algorithm. *Journal of Information Processing and Cybernetics* **27**, 263-271.

Scheithauer, G. (1999):
LP-based bounds for the container and multi-container loading problem. *International Transactions in Operations Research* **6**, 199-213.

Scheithauer, G.; Sommerweiß, U. (1998):
4-Block heuristic for the rectangle packing problem. *European Journal of Operational Research* **108**, 509-526.

Schepers, J. (1997):
A new exact algorithm for general orthogonal d-dimensional knapsack problems. *Lecture Notes in Computer Science* **1284**, 144-156.

Schilling, K. E. (1990):
The growth of m-constraint random knapsacks. *European Journal of Operational Research* **46**, 109-112.

Scholl, A. (1995):
Balancing and sequencing of assembly lines. Physica, Heidelberg.

Scholl, A.; Klein, R.; Jürgens, C. (1997):
BISON: A fast hybris procedure for exactly solving the one-dimensional bin packing problem. *Computers and Operations Research* **24**, 627-645.

Schwerin, P.; Wäscher, G. (1997):
The bin-packing problem: A problem generator and some numerical experiments with FFD packing and MTP. *International Transactions in Operational Research* **4**, 377-389.

Smith, A.; de Cani, P. (1980):
An algorithm to optimize the layout of boxes in pallets. *Journal of the Operational Research Society* **31**, 573-578.

Steudel, H. J. (1979):
Generatine pallet loading patterns: A special case of the two-dimensional cutting stock problem. *Management Science* **25**, 997-1004.

Stoyan, Y. G.; Patsuk, V. N. (2000):
A method of optimal lattice packing of congruent oriented polygons in the plane. *European Journal of Operational Research* **124**, 204-216.

Stoyan, Y. G.; Yaskov, G. N. (1998):
Mathematical model and solution method of optimization problem of placement of rectangles and circles taking into account special constraints. *International Transactions in Operational Research* **5**, 45-57.

Talbot, F. B.; Patterson, J. H.; Gehrlein, W. V. (1986):
A comparative evaluation of heuristic balancing techniques. *Management Science* **32**, 430-454.

Tarasova, T. D.; Rozanova, L. F.; Gabitov, V. A. (1997):
Integer models of rectangular guillotine cutting. In: Kashaeva, Z. G. (Ed.): Decision making under conditions of uncertainty (Cutting-Packing Problems). USATU, Ufa, 125-139.

Terno, J.; Scheithauer, G.; Sommerweiß, U.; Riehme, J. (2000):
An efficient approach for the multi-pallet loading problem. *European Journal of Operational Research* **123**, 372-381.

Thiel, J.; Voss, S. (1994):
Some experiences on solving multiconstraint zero-one knapsack problems with genetic algorithms. *INFOR* **32**, 226-242.

Vance, P. H.; Barnhart, C.; Johnson, E. L.; Nemhauser, G. L. (1994):
Solving binary cutting stock problems by column Generation and branch-and-bound. *Computational Optimization and Applications* **3**, 111-130.

Vanderbeck, F. (2000):
Exact Algorithm for Minimising the Number of Setups in the One-Dimensional Cutting Stock Problems. *Operations Research* **48**, 915-926.

Vanderbeck, F. (2001):
A nested decomposition approach to a three-stage, two-dimensional cutting-stock problem. *Management Science* **47**, 864-879.

Wang, P. Y. (1983):
Two algorithms for constrained two-dimensional cutting stock problems. *Operations Research* **31**, 573-586.

Wäscher, G. (1990):
An LP-based approach to cutting stock problems with multiple objectives. *European Journal of Operational Research* **44**, 175-184.

Wäscher, G.; Gau, T. (1996):
Heuristics for the Integer one-dimensional cutting stock problem: a computational study. *OR Spectrum* **18**, 131-144.

Yanasse, H. H. (1997):
On a pattern sequencing problem to minimize the maximum number of open stacks. *European Journal of Operational Research* **100**, 454-463.

Yuen, B. J. (1995):
Improved heuristics for sequencing cutting patterns. *European Journal of Operational Research* **87**, 57-64.

Yuen, B. J.; Richardson, K. V. (1995):
Establishing the optimality of sequencing heuristics for cutting stock problems. *European Journal of Operational Research* **84**, 590-598.