

# An improved typology of cutting and packing problems

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Received 30 September 2004; accepted 15 December 2005

Available online 19 June 2006

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## Abstract

The number of publications in the area of Cutting and Packing (C&P) has increased considerably over the last two decades. The typology of C&P problems introduced by Dyckhoff [Dyckhoff, H., 1990. A typology of cutting and packing problems. *European Journal of Operational Research* 44, 145–159] initially provided an excellent instrument for the organisation and categorisation of existing and new literature. However, over the years also some deficiencies of this typology became evident, which created problems in dealing with recent developments and prevented it from being accepted more generally. In this paper, the authors present an improved typology, which is partially based on Dyckhoff's original ideas, but introduces new categorisation criteria, which define problem categories different from those of Dyckhoff. Furthermore, a new, consistent system of names is suggested for these problem categories. Finally, the practicability of the new scheme is demonstrated by using it as a basis for a categorisation of the C&P literature from the years between 1995 and 2004.

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*Keywords:* Cutting; Packing; Typology

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## 1. Introduction

A typology is a systematic organisation of objects into homogeneous categories on the basis of a given set of characterising criteria. It is practically oriented and meant to deal with “important” real objects in the first place while hypothetical and/or “less important” real objects might be neglected. A typology resembles a classification, however, with respect to the above-mentioned focus, unlike the latter, it may not be complete (i.e. not all properties of a criterion may be considered explicitly), and it may

be “fuzzy” (i.e. the categories may not always be defined precisely and properly distinguished from each other).

A typology provides a concise view of “relevant”, “important” objects and, thus, prepares the ground for practically oriented research. Furthermore, it helps to unify definitions and notations and, by doing so, facilitates communication between researchers in the field. Finally, if publications are categorised according to an existing typology, it will provide a faster access to the relevant literature.

A typology of OR problems, in particular, provides the basis for a structural analysis of the underlying problem types, the identification and definition of standard problems, the development of models and algorithms, problem generators, etc. For the area of cutting and packing (C&P),

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Dyckhoff (1990) has introduced such a typology, which, unfortunately, has not always found satisfactory international acceptance. Furthermore, almost fifteen years since its initial publication, it became obvious that Dyckhoff's typology is insufficient with respect to the inclusion of current developments. Therefore, the authors decided to present a new, improved typology here, providing a consistent system of problem types which allows for a complete categorisation of all known C&P problems and the corresponding literature. Furthermore, it helps to identify "blank spots", i.e. areas, in which no or only very little research has been done. Finally, the suggested typology is also open to new problem types to be introduced in the future.

For the problem types included in the typology, a straightforward system of notations is introduced. Wherever possible, the problem categories are named in accordance with existing notations. By doing so, the authors do not only intend to avoid unnecessary misinterpretations, or even confusion, but also to improve the degree of acceptance among researchers in the field. The system of notations will also provide the basis for a system of abbreviations which can be used in a database of C&P literature for a simple but precise characterisation. The suggested abbreviations may also become part of a more complex coding scheme of C&P problems.

The remaining part of this paper is organised as follows: In Section 2, the general structure of C&P problems will be presented, not only because the typology to be introduced here addresses C&P problems, but also because the criteria for the definition of the respective problem categories are closely related to elements of this structure. In Section 3, Dyckhoff's typology will be discussed, and, in particular, some severe drawbacks are pointed out which necessitate the development of a new, improved typology. The new typology is outlined in Section 4. In Section 5, criteria and corresponding properties used in the new typology for the definition of problem types are introduced. As far as these criteria have also been used by Dyckhoff, they were complemented by additional, potentially important properties. Then (Section 6), in a first step, two of the criteria, "type of assignment" and "assortment of small objects", are used to define basic problem types. The additional application of a third criterion, "assortment of large objects" leads to intermediate problem types. Finally, the application of the criteria "dimensionality" and "shape of

small items" provides refined problem types. In Section 7 it is demonstrated how well-known problems discussed in the literature fit into the suggested typology. Furthermore, its usefulness and practicality is demonstrated by categorising the literature on C&P problems from the last decade (1995–2004) accordingly. An analysis of recent trends in C&P research, based on the categorisation, is given in Section 8. The paper concludes with an outlook on future work (Section 9), which particularly sketches a potential, more refined scheme for the categorisation of (publications on) C&P problems.

## 2. Structure of cutting and packing problems

Cutting and packing problems have an identical structure in common which can be summarised as follows:

Given are two sets of elements, namely

- a set of large objects (input, supply) and
- a set of small items (output, demand),

which are defined exhaustively in one, two, three or an even larger number ( $n$ ) of geometric dimensions. Select some or all small items, group them into one or more subsets and assign each of the resulting subsets to one of the large objects such that the *geometric condition* holds, i.e. the small items of each subset have to be laid out on the corresponding large object such that

- all small items of the subset lie entirely within the large object and
- the small items do not overlap,

and a given (single-dimensional or multi-dimensional) objective function is optimised. We note that a solution of the problem may result in using some or all large objects, and some or all small items, respectively.

Formally, five sub-problems can be distinguished, which, of course, have to be solved simultaneously in order to achieve a "global" optimum:

- selection problem regarding the large objects,
- selection problem regarding the small items,
- grouping problem regarding the selected small items,
- allocation problem regarding the assignment of the subsets of the small items to the large objects,

- layout problem regarding the arrangement of the small items on each of the selected large objects with respect to the geometric condition.

Special (types of) C&P problems are characterised by additional properties. In particular, they may turn out to be degenerated in the sense that they do not include all of the above-mentioned sub-problems.

### 3. Dyckhoff's typology

Fig. 1 summarises the main features of Dyckhoff's typology. The author introduces four criteria (characteristics) according to which C&P problems are categorised. Criterion 1, dimensionality, captures the minimal number ( $1, 2, 3, n > 3$ ) of geometric dimensions which are necessary to describe the required layouts (patterns) completely. Regarding criterion 2, the kind of assignment of small items to large objects, Dyckhoff distinguishes two cases, indicated by B and V. B stands for the German "*Beladeproblem*", meaning that all large objects have to be used. A selection of small items has to be assigned to the large objects. V (for the German "*Verladeproblem*") characterises a situation in which all small items have to be assigned to a selection of large objects. Criterion 3 represents the assortment of the large objects. O stands for one large object, I for several but identical large objects,

and D for several different large objects. Criterion 4 likewise characterises the assortment of the small items. Dyckhoff distinguishes assortments consisting of few items (F), many items of many different figures (M), many items of relatively few different figures (R), and congruent figures (C; we will be using the notion "shape" here, instead of "figure").

When it was initially published, Dyckhoff's work represented a milestone in C&P research as it highlighted the common underlying structure of cutting problems on one hand and packing problems on the other. By doing so, it supported the integration and cross-fertilisation of two largely separated research areas. Unfortunately, on an international level his typology has not always been accepted as widely as was desirable, most probably due to the fact that the provided coding scheme was not self-explanatory from the view point of an international (English-speaking) community of researchers. This becomes particularly evident for the problem types B and V.

Obviously, this deficiency could be overcome by introducing more appropriate (English) names for the respective problem categories/types. However, some drawbacks have to be taken more seriously, which became evident in the light of some recent developments in the field of C&P.

- Not necessarily all C&P problems (in the narrow sense) can be assigned uniquely to problem types

<p>1. Dimensionality          (1) one-dimensional          (2) two-dimensional          (3) three-dimensional          (N) N-dimensional with <math>N &gt; 3</math></p> <p>2. Kind of assignment          (B) all objects and a selection of items          (V) a selection of objects and all items</p> <p>3. Assortment of large objects          (O) one object          (I) identical figure          (D) different figures</p> <p>4. Assortment of small items          (F) few items (of different figures)          (M) many items of many different figures          (R) many items of relatively few different (non-congruent) figures          (C) congruent figures</p>
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Fig. 1. Categorisation criteria, main types, and coding scheme of Dyckhoff's typology (cf. Dyckhoff, 1990, p. 154).

In Dyckhoff's paper already, a well-known standard problem, the *Vehicle Loading Problem*, has been coded both as 1/V/I/F and 1/V/I/M (Dyckhoff, 1990, p. 155). F characterises a situation in which few items of different shapes (i.e. items which are different with respect to shape, size and/or orientation) are to be assigned. M represents a situation with many items of many different shapes. Apart from the fact that it is definitely desirable to have only one option available for the assignment of a well-known standard problem like the Vehicle Loading Problem, it does not become clear how this differentiation will provide a set of problem categories which is more homogeneous (with respect to model building and the development of algorithms) than a single type, which includes both categories. (It is interesting to note that problem category F only appears once in Dyckhoff's examples of combined types (1990, p. 155). One may take this as an additional indicator that this kind of differentiation might not prove very promising in the end.)

- Dyckhoff's typology is partially inconsistent; its application might have confusing results

In the literature, a particular two-dimensional packing problem, the so called “*Strip-Packing Problem*”, namely the packing of a set of small items (often rectangles) of different sizes into a single rectangle with fixed width and minimal (variable) length (Jacobs, 1996; Martello et al., 2003), has attracted considerable attention. Most probably, this problem would be coded both by researchers and by practitioners as 2/V/O/M, while in Dyckhoff's typology it has been assigned the notation 2/V/D/M. Dyckhoff justifies this notation – rather artificially from our point of view – by saying that this problem “. . . is equivalent to an assortment selection problem where the stock is given by an infinite number of objects of this width and of all possible lengths and where only one object has to be chosen from stock, namely that of minimal length” (1990, p. 155). What makes Dyckhoff's characterisation of the problem even more confusing is the fact that he calls it a *Two-Dimensional Bin Packing Problem* (Dyckhoff, 1990, p. 155). Not only in our understanding (also see Lodi et al., 2002a; Miyazawa and Wakabayashi, 2003; Faroe et al., 2003) it would have been more obvious to reserve this name for the natural extension of the *Classic (One-Dimensional) Bin Packing Problem*, i.e. the packing of a set of small items of different sizes into a minimum number of rectangles (large objects) of identical size. The notation becomes even more questionable for two-dimensional problems where both width and length are variables, and, likewise, for three-dimensional problems, where width, length and/or height are variables. We conclude that – in order to avoid confusion about the contents of problem notations and definitions – it is advisable to distinguish problem types with respect to fixed and with respect to variable dimensions.

- Application of Dyckhoff's typology does not necessarily result in homogeneous problem categories

Gradišar et al. (2002) notice that in the case of one-dimensional cutting, where a large number of small items of relatively few different shapes has to be produced from standard material in unlimited supply, two situations should be distinguished if the standard material comes in different sizes. In the first situation, the large objects can be separated into a few groups of identical size, while in the second sit-

uation all large objects are entirely different. In Dyckhoff's coding scheme both situations would be included in the problem type 1/V/D/R. Gradišar et al. (2002, p. 1208) now argue that the inclusion in the same problem category is not very useful because the two situations require different solution approaches, namely a pattern-oriented approach for cutting problems with few groups of identical large objects and an item-oriented approach for such problems with entirely different large objects. We just would like to add that the same can be observed for problems of higher dimensions and for packing problems, as well. Therefore, we conclude that, in order to develop categories of homogeneous problems, the properties (main types) of Dyckhoff's third criterion, the assortment of large objects, should be further differentiated.

The latter aspect turns out to represent the most severe limitation of Dyckhoff's typology, as it generally questions whether one of the central goals of the introduction of a problem typology is achieved, namely to provide a homogeneous basis for the development of models and algorithms.

#### 4. Outline of the new typology and overview of problem categories

Problem types can generally be defined as elementary types or combined types. Types which can be used as a basis for the development of models, algorithms, and problem generators, and for the categorisation of literature must be relatively homogeneous. Consequently, it is very likely that they must be introduced as combined types, which stem from the subsequent or simultaneous application of different categorisation criteria. Five criteria will be used here for the definition of combined problem types of C&P problems, namely “dimensionality”, “kind of assignment”, “assortment of large objects”, “assortment of small items”, and “shape of the small items”.

It goes without saying that each typology of C&P problems should offer an option to characterise a given problem with respect to the number of problem-relevant dimensions. Consequently, this criterion will be adopted directly from Dyckhoff's typology.

Dyckhoff's criterion “kind of assignment” has proven to be useful in distinguishing between different kinds of C&P problems in the past. Therefore, it will also be used here; however, in order to avoid the

German notations “Verladeproblem” and “Beladeproblem”, the authors will refer to the corresponding problem categories/elementary types – in a general sense – as “input minimisation” and “output maximisation”, respectively.

On the other hand, in order to overcome the above-sketched limitations of Dyckhoff’s typology, the two criteria “assortment of large objects” and “assortment of small items” will be redefined and/or supplemented with new properties.

Unlike in Dyckhoff’s typology, the two criteria “kind of assignment” and “assortment of small items” will not be taken for the definition of two different fields in a classification scheme for C&P problems. Instead, they will be used in combination in order to define *basic problem types*. These basic problem types (which already represent combined types in the sense of Dyckhoff; see Dyckhoff, 1990, p. 154) provide the core objects for the introduction of a new, more widely accepted nomenclature. Existing names have been adopted as far as possible, in particular, wherever there was no or only a small probability that their use would result in misinterpretations of their contents.

The subsequent application of the criterion “assortment of large objects” will provide *intermediate problem types*. Further characterisation with respect to the number of problem relevant dimensions (“dimensionality”) and – in the case of problems of two and more dimensions – with respect to the “shape of small items” will provide *refined (combined) problem types*. The name of each of these refined types consists of the name of the underlying intermediate type and one or two additional adjectives which indicate the respective properties. Refined problem types will be used for the categorisation of publications, here.

An instance of a specific refined problem type will exhibit all the properties which have been used for the definition of the respective category and probably additional constraints and/or characteristics. A problem instance which only exhibits the defining properties, but no additional constraints or characteristics could be interpreted as being (an instance of) a *(first-level) standard problem (type)*. *First-level non-standard problems* (problem types) are characterised by the properties defining the respective problem category and additional constraints and/or characteristics. Of course, in particular when considered in scientific research, they may represent well-known standard problems as well. In such case, in order to distinguish them from the previously

mentioned ones, we would call them *second-level standard problems* whenever necessary. The identification of standard problems (types) represents a major goal of our typology. These problem types provide the basis of scientific research, in particular for the development of (standard) models, algorithms, and problem generators.

Fig. 2 gives an overview of the previously introduced types and their relationships. In this paper, we concentrate on “pure” C&P problem types in the above-defined sense (see Section 2), i.e. problems in which the solution consists of information on the set of patterns according to which the (selected) small items have to be cut from/packed into (a subset of) the large objects and the corresponding objective function value. The inclusion of additional aspects which extend the view of the planning problem beyond the core of cutting or packing will give rise to an *extended problem type* or a *problem extension*. Apart from information on the patterns, a solution to a problem of this kind includes additional information on other problem-relevant aspects such as the number of different patterns (as in the pattern minimisation problem; cf. Vanderbeck, 2000), processing sequences (as in the pattern sequencing problem; cf. Foerster and Wäscher, 1998; Yanasse, 1997; Yuen, 1995; Yuen and Richardson, 1995) or lot-sizes (cf. Nonås and Thorstenson, 2000). We would like to point out, however, that the “C&P-related part” of problems of this type can be categorised in the same way as “pure” problems.

When defining basic, intermediate and refined problem types, it will be necessary to introduce certain assumptions. Some of these assumptions are related to general properties of the problem and restrict the view to single-objective, single-period and deterministic problems. Replacing the assumptions by different ones leads to problem types which will be considered as *problem (type) variants*, here. Problems with multiple objectives (cf. Wäscher, 1990), stochastic problems (in which, e.g., the sizes of the large objects are random variables; cf. Das and Ghosh, 2003), fuzzy problems (where the costs of the large objects are fuzzy coefficients; cf. Katagiri et al., 2004), or on-line problems (in which, e.g., parcels arrive one after another at a packing station and a decision has to be made immediately on their arrival where they should be packed into a container; cf. Hemminki et al., 1998; Abdou and Elmasry, 1999) belong to this category. A second class of assumptions also leading to problem variants arises from C&P-specific

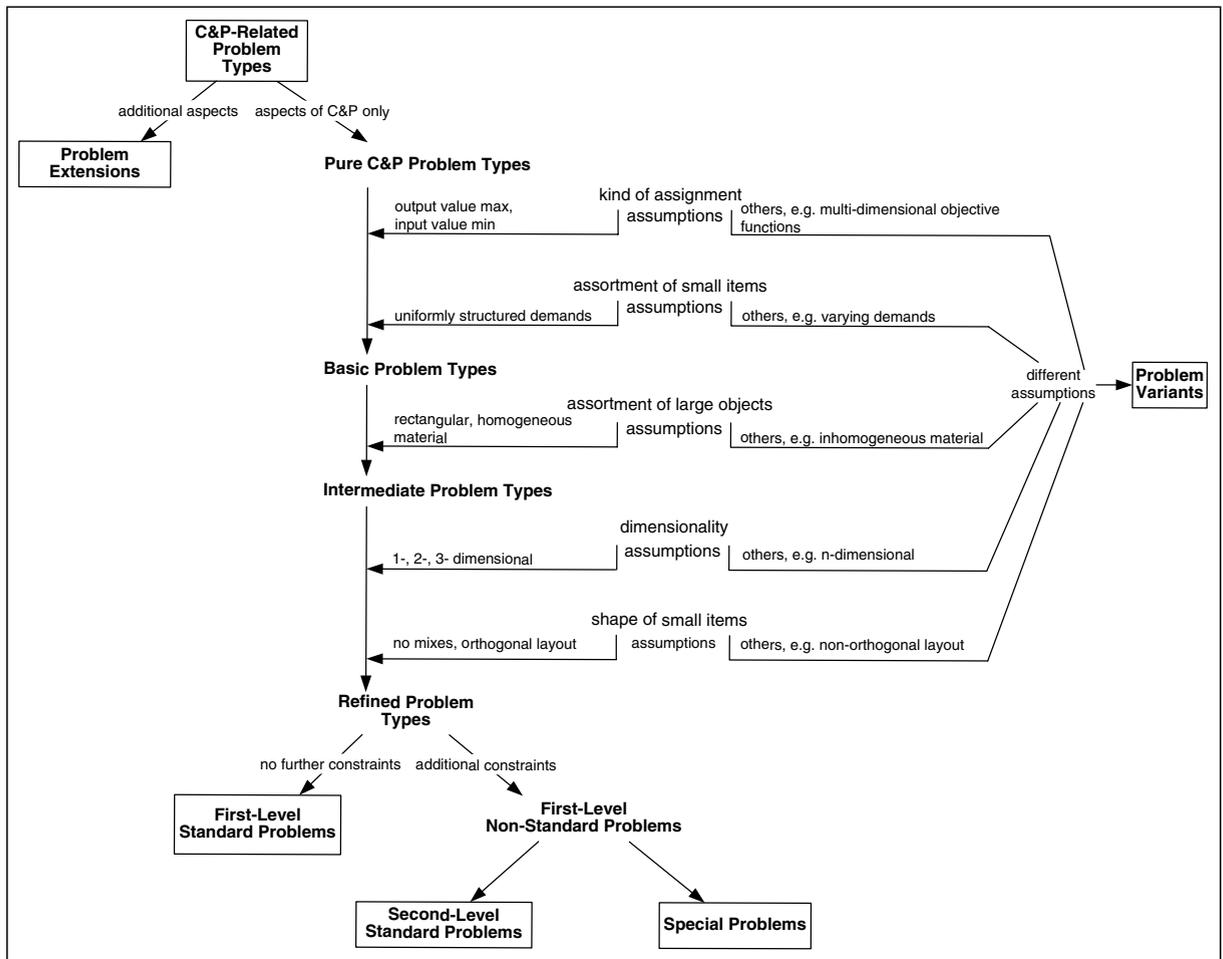


Fig. 2. Overview of problem types related to C&P problems.

aspects. They will be explained in greater detail later.

**5. (Modified) criteria for the definition of problem types**

*5.1. Dimensionality*

We distinguish between *one-*, *two-*, and *three-dimensional* problems. In the literature, occasionally, also problems with more than three geometric dimensions are considered (e.g., Lins et al., 2002). Problems of this type ( $n > 3$ ) are looked upon as variants, here.

*5.2. Kind of assignment*

Again, as in Dyckhoff (1990), we introduce two basic situations, of which we prefer to speak of *out-*

*put (value) maximisation* and *input (value) minimisation*, respectively.

- output (value) maximisation

In the case of output (value) maximisation, a set of small items has to be assigned to a given set of large objects. The set of large objects is not sufficient to accommodate all the small items. All large objects are to be used (in other words: there is no selection problem regarding the large objects), to which a selection (a subset) of the small items of maximal value has to be assigned.

- input (value) minimisation

Again, a given set of small items is to be assigned to a set of large objects. Unlike before, in the case of input (value) minimisation the set of large objects

is sufficient to accommodate all small items. All small items are to be assigned to a selection (a subset) of the large object(s) of minimal “value”. There is no selection problem regarding the small items.

Here, “output (value) maximisation” and “input (value) minimisation” are used in a general, non-specific manner. When treating specific problems, the “value” of objects/items has to be defined more precisely and may be represented by costs, revenues, or material quantities. Often, the value of the objects/items can be assumed to be directly proportional to their size such that the objective function considers length (one-dimensional problems), area (two-dimensional problems), or volume (three-dimensional problems) maximisation (output) or minimisation (input). In such cases, it might also be possible to translate both “output (value) maximisation” and “input (value) minimisation” into “waste minimisation”, i.e. the minimisation of the total size of unused parts of the (selected) large objects. In the environment of cutting problems often the term “trim-loss minimisation” is used.

We would also like to point out that – in order to define basic problem types – only these two situations will be considered here. Of course, problems encountered in practice and/or discussed in the literature may be characterised by the fact that a selection problem exists with respect to both large objects and small items. This requires an extended objective function which combines revenues and costs (“profit maximisation”). Also situations exist in which more than one objective function may have to be taken into account. Again, problems of this type will be considered as problem variants here.

### 5.3. Assortment of small items

With respect to the assortment of the small items we distinguish three cases, namely *identical small items*, a *weakly heterogeneous assortment of small items*, and a *strongly heterogeneous assortment of small items* (cf. Fig. 3):

- identical small items

Regarding their problem-relevant dimensions (i.e. their extension in the problem-relevant number of dimensions “length”, “width”, and “height”), all

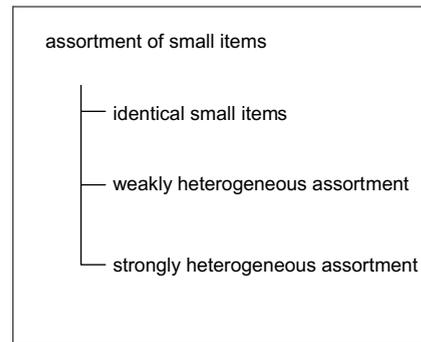


Fig. 3. Cases to be distinguished with regard to the assortment of small items.

items are of the same shape and size. In the output maximisation case, it can be assumed that the (single) item type has an unlimited demand. This problem category is identical with Dyckhoff’s elementary type C (“congruent shapes”; cf. Dyckhoff, 1990, p. 154).

- weakly heterogeneous assortment

The small items can be grouped into relatively few classes (in relation to the total number of items), for which the items are identical with respect to shape and size. By definition, small items of identical shape and size which require different orientations are treated as different kinds of items. The demand of each item type is relatively large, and may or may not be limited by an upper bound. This category corresponds to Dyckhoff’s elementary type R (“many items of relatively few different (non-congruent) shapes”; cf. Dyckhoff, 1990, p. 154).

- strongly heterogeneous assortment

The set of small items is characterised by the fact that only very few elements are of identical shape and size. If that occurs, the items are treated as individual elements. Consequently, the demand of each item is equal to one. This category includes Dyckhoff’s elementary types M (“many items of many different shapes”) and F (“few items (of different shapes)”; cf. Dyckhoff, 1990, p. 154).

For the definition of standard problems we assume that the set of small items is uniformly structured, i.e. that it does not contain items with large

demands and others with small demands. A problem with strongly varying demands (cf. Riehme et al., 1996) will be considered as a variant, here.

#### 5.4. Assortment of large objects

With respect to the assortment of the large objects we introduce the following cases (cf. Fig. 4):

- one large object

In this case the set of large objects consists of a single element. The extension of the large object may be fixed in all problem-relevant dimensions (“*all dimensions fixed*”), or its extension may be variable in one or more dimensions (“*one or more variable dimensions*”). The first category is identical with Dyckhoff’s type O, while the second category represents an extension of Dyckhoff’s set of elementary types (cf. Dyckhoff, 1990, p. 154).

- several large objects

With respect to the kind of problems which are described in the literature, in the case of several large objects it does not appear necessary to distinguish between fixed and variable dimensions; only fixed dimensions will be considered. In analogy to the categories which have been introduced for the assortment of the small items, we distinguish between *identical large objects*, a *weakly* and a *strongly heterogeneous assortment of large objects*. By doing so, we again extend Dyckhoff’s typology, who only identifies large objects with identical (type I) and different shapes (type D).

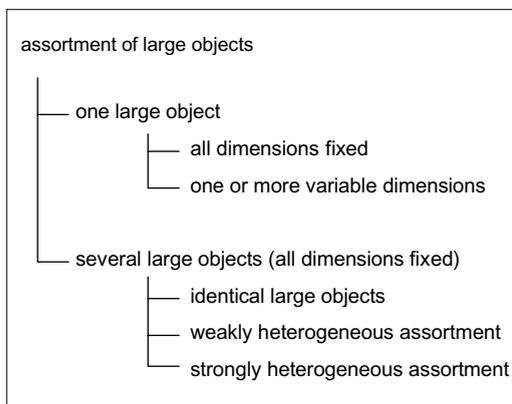


Fig. 4. Cases to be distinguished with regard to the assortment of large objects.

For the definition of basic problem types we assume that – in the two- and three-dimensional case – all large objects are of rectangular shape (rectangles, cuboids) and consist of homogeneous material. Non-rectangular large objects (e.g., circular objects such as discs) and/or non-homogeneous large objects (e.g., stock material including defects) give rise to problem variants, again.

#### 5.5. Shape of small items

In the case of two- and three-dimensional problems, for the definition of refined problem types we further distinguish between *regular small items* (rectangles, circles, boxes, cylinders, balls, etc.) and *irregular* (also called: *non-regular*) ones. In the two-dimensional case, the former are sometimes further distinguished into *rectangular items*, *circular items*, and others.

In accordance with what is usually considered in the literature, we assume that rectangular items are to be laid out orthogonally. Furthermore, the set of small items either consists of regular or irregular elements. Problems which allow for non-orthogonal layouts and/or mixes of regular and irregular small items will be looked upon as problem variants again.

## 6. Basic, intermediate and refined problem types

### 6.1. Basic problem types

Basic types of C&P problems are developed by combination of the two criteria “type of assignment” and “assortment of small items”. Fig. 5 depicts the relevant combinations and the corresponding basic problem types.

In the following sections these problem types will be characterised in greater detail.

#### 6.1.1. Output maximisation types

Problems of the output maximisation type have in common that the large objects are only supplied in limited quantities which do not allow for accommodating all small items. As the value of the accommodated items has to be maximised, all large objects will be used. In other words, generally there is a selection problem regarding the small items, but none regarding the large objects.

According to Fig. 5, we distinguish the following (basic, output maximisation) problem types:

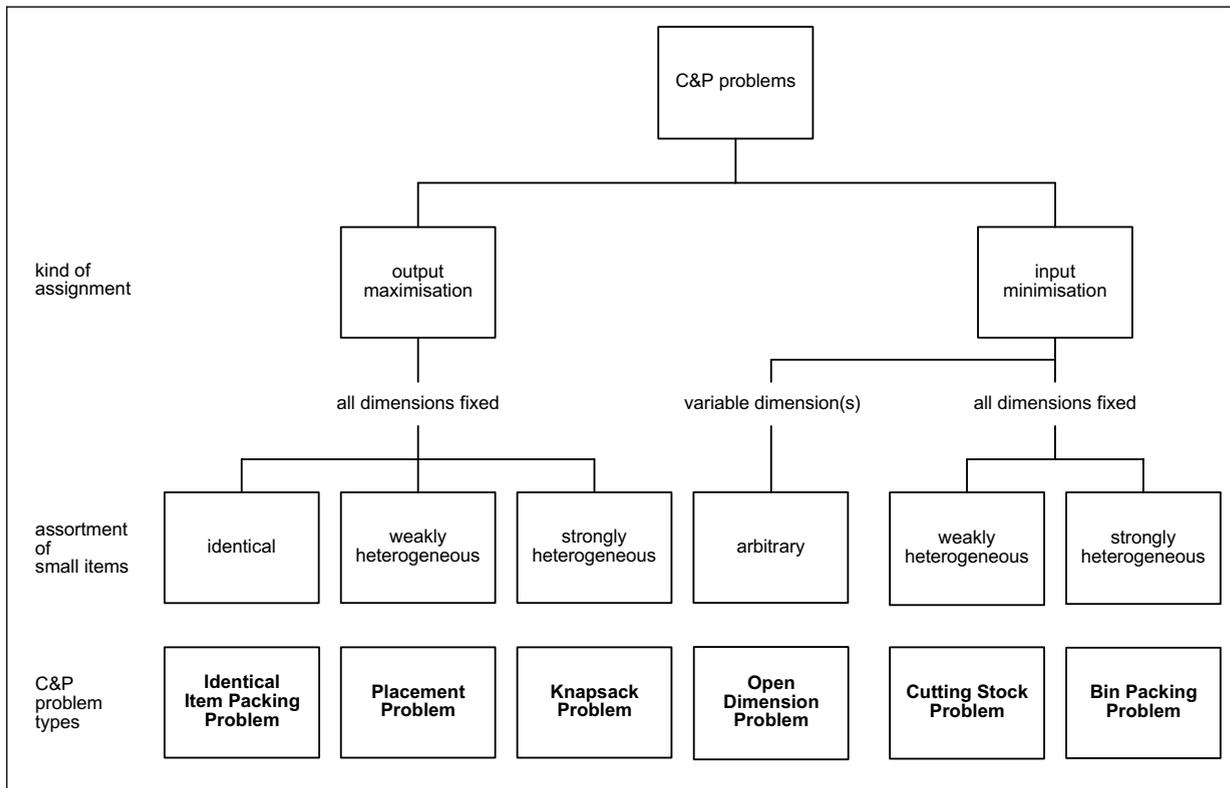


Fig. 5. Basic problem types.

- *Identical Item Packing Problem*

This problem category consists of the assignment of the largest possible number of identical small items to a given, limited set of large objects. We note that, due to the fact that all the small items are identical, there is in fact no real selection problem regarding the small items, and, furthermore, neither a grouping nor an allocation problem occurs. In other words, the general structure of C&P problems (cf. Section 2) is reduced to a layout problem regarding the arrangement of the (identical) small items on each of the large objects with respect to the geometric condition.

- *Placement Problem*

In the literature, problems of this category are known under many different names. In order to avoid additional confusion, here we have introduced a somewhat more neutral notion. Here, the term “Placement Problem” defines a problem category in which a weakly heterogeneous assortment of small items has to be assigned to a given,

limited set of large objects. The value or the total size (as an auxiliary objective) of the accommodated small objects has to be maximised, or, alternatively, the corresponding waste has to be minimised.

- *Knapsack Problem*

According to our interpretation, the Knapsack Problem represents a problem category which is characterised by a strongly heterogeneous assortment of small items which have to be allocated to a given set of large objects. Again, the availability of the large objects is limited such that not all small items can be accommodated. The value of the accommodated items is to be maximised.

### 6.1.2. Input minimisation types

Problems of the input minimisation type are characterised by the fact that the supply of the large objects is large enough to accommodate all small items. Their demands have to be satisfied completely, so that no selection problem regarding the small items exists. The value of the large objects necessary to accommodate all small items has to be minimised.

• *Open Dimension Problem*

The Open Dimension Problem defines a problem category in which the set of small items has to be accommodated completely by one large object or several large objects. The large objects are given, but their extension in at least one dimension can be considered as a variable. In other words, this problem involves a decision on fixing the extension(s) in the variable dimension(s) of the large objects. Only the part(s) of the large object(s) necessary to accommodate the items completely represents input within the meaning of the general structure of C&P problems (cf. Section 2). The value of the input (or a corresponding auxiliary measure like length, size, or volume) is to be minimised. For the definition of basic problem types we restrict ourselves to large objects which – before and after having fixed their extension(s) in the variable dimension(s) – are rectangles (two-dimensional problems) or cuboids (three-dimensional problems). By doing so, in particular those problems are excluded from our analysis in which the small items have to be enclosed in non-rectangular large objects of minimal size (e.g., when circles have to be packed into another circle of minimal radius; cf. Birgin et al., 2005, pp. 27ff.), or in which the density of the packing has to be maximised (e.g., as in the case of two-dimensional lattice packing; cf. Stoyan and Patsuk, 2000).

• *Cutting stock problem*

Problems of this category require that a weakly heterogeneous assortment of small items is completely allocated to a selection of large objects of minimal value, number, or total size. The extension of the large objects is fixed in all dimensions. We point out that we do not make any assumptions on the assortment of the large objects. It may consist of identical objects, but it could also be a weakly or strongly heterogeneous assortment.

• *Bin packing problem*

In contrast to the previously described problem category, this one is characterised by a strongly heterogeneous assortment of small items. Again, the items have to be assigned to a set of identical large objects, a weakly heterogeneous or strongly heterogeneous assortment of large objects. The value, number, or total size of the necessary large

objects (or another corresponding auxiliary objective) has to be minimised.

6.2. *Intermediate problem types*

In order to define more homogeneous problem types the above-developed basic problem types are structured further into intermediate problem types. This is achieved by taking into account the assortment of the large objects as an additional differentiating criterion. Figs. 6 and 7 depict the intermediate problem types related to output maximisation, Figs. 8 and 9 the intermediate types related to input minimisation. Figs. 6–9 also present our suggestions for naming these types.

We note at this stage that – due to its simple problem structure – it is not necessary to further differentiate the Identical Item Packing Problem. In order to solve a problem of this type, it can be split into a set of independent sub-problems where each sub-problem is related to a particular large object (or a particular type of large objects, if the large objects are at least partially identical) to which the

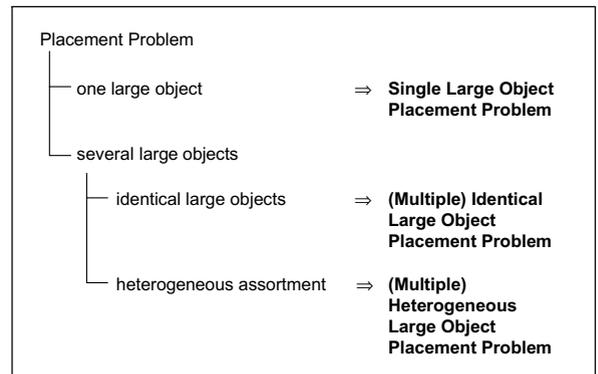


Fig. 6. Intermediate types of the placement problem.

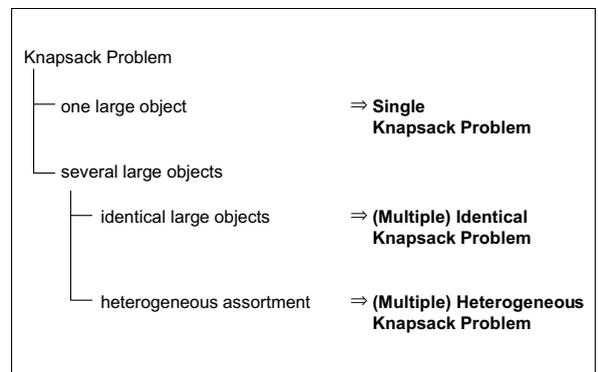


Fig. 7. Intermediate types of the knapsack problem.

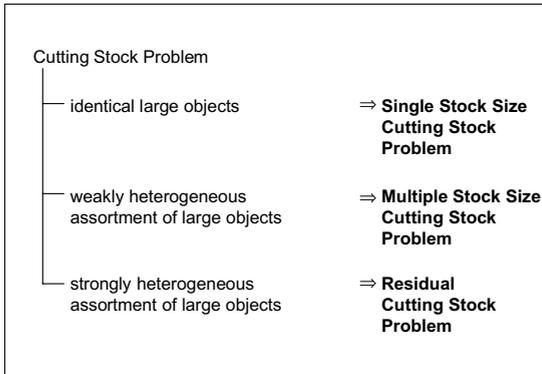


Fig. 8. Intermediate types of the cutting stock problem.

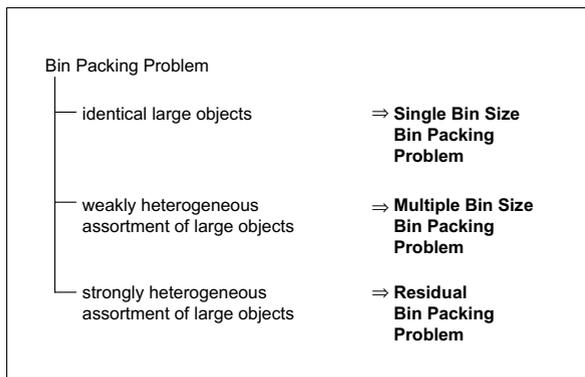


Fig. 9. Intermediate types of the bin packing problem.

largest possible number of small items has to be assigned. Consequently, in this typology the name Identical Item Packing Problem will also be reserved for a (combined) problem type in which the largest possible number of identical small items has to be assigned to a single given large object. Furthermore, with respect to the fact that the existing literature on the Open Dimension Problem concentrates on a very limited number of standard problems, we also refrained from structuring this basic problem type in greater detail at this stage.

Figs. 10 and 11 summarise the system of intermediate problem types which has been introduced here. They can be interpreted as the “landscape” of C&P problems. Also, for each intermediate problem type, an abbreviation derived from the corresponding name is given which allows for a unique identification of each type.

6.3. Refined problem types

In a final step, refined problem types are obtained by application of the criterion “dimensionality” and – for two- and three-dimensional problems – of the criterion “shape of the small items”. The resulting subcategories are characterised by adjectives which are added to (the names of) the intermediate problem types (IPT) according to the following system:

characteristics of the large objects		assortment of the small items		
		identical	weakly heterogeneous	strongly heterogeneous
all dimensions fixed	one large object	Identical Item Packing Problem IIPP	Single Large Object Placement Problem SLOPP	Single Knapsack Problem SKP
	identical	X	Multiple Identical Large Object Placement Problem MILOPP	Multiple Identical Knapsack Problem MIKP
	heterogeneous		Multiple Heterogeneous Large Object Placement Problem MHLOPP	Multiple Heterogeneous Knapsack Problem MHKP

Fig. 10. Landscape of intermediate problem types: output maximisation.

characteristics of large objects		assortment of small items		weakly heterogeneous	strongly heterogeneous
		all dimensions fixed	identical	Single Stock Size Cutting Stock Problem SSSCSP	Single Bin Size Bin Packing Problem SBSBPP
weakly heterogeneous	Multiple Stock Size Cutting Stock Problem MSCSP		Multiple Bin Size Bin Packing Problem MBSBPP		
strongly heterogeneous	Residual Cutting Stock Problem RCSP		Residual Bin Packing Problem RBPP		
one large object variable dimension(s)		Open Dimension Problem ODP			

Fig. 11. Landscape of intermediate problem types: input minimisation.

{1, 2, 3}-dimensional {rectangular, circular, . . . , irregular} {IPT}.

**7. Specific problem categories**

In order to make the consequences of the suggested typology clearer, we will now explain the different categories in further detail by giving examples of specific cutting and packing problems (described in the literature) which belong to the problem types introduced above. In particular, well-known standard problems will be pointed out as representatives of the respective categories.

*7.1. Identical Item Packing Problem*

A well-known (regular/rectangular) representative of this problem type is the (*Classic Manufacturer’s Pallet Loading (Packing) Problem* (MPLP; Dowsland, 1987; Morabito and Morales, 1998), in which a single pallet has to be loaded with a maximal number of identical boxes (the problem itself – without this particular name – has been described even earlier in the literature, see, e.g., Steudel, 1979;

Smith and de Cani, 1980). It is usually assumed that the boxes are loaded in layers, in which all boxes have the same vertical orientation. By means of this assumption, the problem is actually reduced to a two-dimensional one (cf. Bischoff and Ratcliff, 1995, p. 1322; also see Dyckhoff’s (1990, p. 155) characterisation of the problem as being of type 2/B/O/C), namely to the problem of assigning a maximal number of small identical rectangles (representing the “bottom” surfaces of the boxes) to a given large rectangle (representing the pallet). In other words: The MPLP is a two-dimensional, rectangular Identical Item Packing Problem (IIPP). The *Cylinder Packing Problem* (Correia et al., 2000, 2001; Birgin et al., 2005) possesses an almost identical problem structure, in which – instead of identical rectangles – a maximal number of non-overlapping circles of the same size has to be assigned to a given rectangle (two-dimensional, circular IIPP; also see Isermann, 1991). Among others, it can be found in the production of cans where the circles represent the lids and bottoms of (cylindrical) cans to be cut from tin plates, or in logistics management, where cylindrical cans have to be packed on a pallet.

The *Single-Box-Type Container Packing Problem* (as discussed in George, 1992) is an example of a three-dimensional special case of the Rectangular (i.e. regular) IIPP. It requires the loading of a single container (large object) with a maximum number of identical boxes (small items).

### 7.2. Single Large Object Placement Problem

The *Bounded Knapsack Problem* and the *Unbounded Knapsack Problem* (as described in Martello and Toth, 1990, pp. 82–91, and pp. 91–103, respectively) are one-dimensional representatives of the SLOPP type. Both require that a (single) knapsack (large object) of given limited weight capacity has to be packed with a selection (subset) from a given set of small items of given weights and values, such that the value of the packed items is maximised. The assortment of the small items is weakly heterogeneous, only relatively few different item types can be identified. The number of times an item of a particular type can be packed may be limited (bounded problem), or unlimited (unbounded problem).

Haims and Freeman (1970) introduce a two-dimensional problem of the SLOPP type which they name *Template-Layout Problem*. According to their general definition, a weakly heterogeneous set of regular or non-regular small items (“forms”) has to be cut from a single large rectangle such that the value of the cut items is maximised. In the subsequent discussion, the authors concentrate on rectangular small items (i.e. the discussed problem is of the two-dimensional, rectangular SLOPP type). In Christofides and Whitlock, 1977; Beasley, 1985a; Christofides and Hadjiconstantinou, 1995, e.g., the authors also restrict themselves to rectangular small items. They call it the *Two-Dimensional Cutting Problem* (more precisely, we would rather call it the two-dimensional, rectangular SLOPP). The number of times an item of a particular type can be cut from the large rectangle may be constrained explicitly by an upper bound (as in Christofides and Whitlock, 1977; Wang, 1983; Beasley, 1985b; Christofides and Hadjiconstantinou, 1995; Beasley, 2004). In Scheithauer and Sommerweiß (1998) the unbounded version of this problem (also see Herz, 1972; Beasley, 1985c) is discussed under the name *Rectangle Packing Problem*. In the (*Constrained*) *Circular Cutting Problem* (Hifi and M’Hallah, 2004; according to the suggested typology, the problem would be characterised as a two-dimensional,

circular SLOPP), a rectangular plate (large object) of given size has to be cut down into circular items, which are of  $m$  different types (radii). The number of small items which may be cut is bounded, and the objective is to minimise the unused space of the plate.

What has been called the *Single Container Loading (Packing) Problem* in the literature (George and Robinson, 1980; Bortfeldt et al., 2003) is an example of the three-dimensional, rectangular SLOPP. It requires loading a fairly large, weakly heterogeneous consignment of boxes into a given container such that the volume or value of the packed boxes is maximised, or, equivalently, the unused space of the container or the value of the unpacked boxes is minimised (also see Ratcliff and Bischoff, 1998; Bortfeldt and Gehring, 1998). A set of weakly heterogeneous cargo has to be packed on a pallet in the *Distributor’s (Single) Pallet Loading Problem* (Hodgson, 1982; Bischoff et al., 1995). Unlike the Manufacturer’s Pallet Loading Problem, this is a truly three-dimensional rectangular problem, as – due to the different box sizes – it may not be sufficient to concentrate on layered packing if the space available above the pallet is to be used in the best possible way. In real-world Single Container and Pallet Loading Problems, the number of boxes which are to be packed of a particular box type will be limited by an upper bound. Hifi (2004), on the other hand, considers the unconstrained case.

### 7.3. Multiple Identical Large Object Placement Problem

Straightforward extensions of the single large object placement problem could take into account multiple identical large objects. This could give rise, e.g., to the (Bounded) Multiple Knapsack Problem, the Multiple Stock Sheet Cutting Problem, etc. Problems of this type (MILOPP), however, have not yet been discussed in the literature, at least not to the knowledge of the authors of this paper.

### 7.4. Multiple Heterogeneous Large Object Placement Problem

Another extension of the Single Large Object Placement Problem which takes into account multiple, non-identical large objects also appears to having been treated very rarely in the literature. A one-dimensional cutting problem is described in

Gradišar et al. (1999, p. 559, case 2), in which a weakly heterogeneous assortment of order lengths has to be cut from a set of input lengths which are all different (one-dimensional MHLOPP). The supply of input lengths is not sufficient to satisfy all orders. Therefore, the objective is to minimise the total size of the uncut order lengths.

Eley (2003) considers three-dimensional container loading problems, in which a weakly heterogeneous consignment of boxes has to be packed into containers available in different sizes. In one of the discussed situations, the available container space is not sufficient to accommodate all boxes. Therefore, a selection of boxes has to be determined that maximises the volume utilisation, or, alternatively, the value of the packed boxes (three-dimensional, rectangular MHLOPP). An exact algorithm for the unconstrained version of this problem is introduced in Hifi (2004).

### 7.5. Single Knapsack Problem

The *Classic (One-Dimensional) Knapsack Problem*, also called *0-1-Knapsack Problem* (Martello et al., 2000; Martello and Toth, 1990, p. 13) which requires packing a given set of (different) items of given weights and values into a (single) knapsack of given limited weight capacity such that the value of the packed items is maximised (cf. Martello and Toth, 1990, pp. 3, 13–77) obviously is a special case of this problem type (i.e. the one-dimensional SKP). The *Subset-Sum Problem* (cf. Martello and Toth, 1990, pp. 105ff.) is a 0-1-knapsack problem in which the weight of an item is identical with its value. A straightforward extension of this problem, namely the *Multiconstraint (0–1) Knapsack Problem* (Gavish and Pirkul, 1985; Drexl, 1988; Thiel and Voss, 1994, the problem is also called *m-Constraint Knapsack Problem*, cf. Schilling, 1990), also belongs to this category. Apart from the capacity constraint,  $m - 1$  additional constraints have to be satisfied by the packed items.

Extensions of the *Classic (One-Dimensional) Knapsack Problem* into two and more geometric dimensions give rise to the *Two-Dimensional (Single Orthogonal) Knapsack Problem* (Caprara and Monaci, 2004; also see e.g., Healy and Moll, 1996), in which a set of small, distinct rectangles has to be cut from a single large rectangle (two-dimensional, rectangular SKP), and the *Three-Dimensional (Single Orthogonal) Knapsack Problem* (also called *Knapsack Container Loading Problem*; cf. Pisinger,

2002), in which rectangular-shaped boxes have to be packed into a container (three-dimensional, rectangular SKP; also see Bischoff and Marriott, 1990; Scheithauer, 1999; Bortfeldt and Gehring, 2001). In both cases usually the value of the cut/packed small items is to be maximised. If their value can be assumed to be proportional to their size/volume, then equivalently the unused space of the large rectangle or the container can be minimised. Fekete and Schepers (1997) finally look at the *n-Dimensional (Single Orthogonal) Knapsack Problem*, which extends the *Classic Knapsack Problem* into  $n$  geometric dimensions.

George et al. (1995) describe a two-dimensional, circular SKP, in which (the bottom of) a container (rectangle) has to be filled with a set of distinct pipes (circles) such that the value of the selected pipes is maximised.

### 7.6. Multiple Identical Knapsack Problem

In the literature, a specific one-dimensional case of the *Multiple Identical Knapsack Problem* is known as the *Maximum Cardinality Bin Packing Problem*, in which a fixed number of large objects with a given, identical capacity and a (strongly heterogeneous) set of small, indivisible items of given weights are given. The objective is to maximise the number of packed items (Labbé et al., 2003).

The *Multiple Container Packing Problem* (as described in Raidl and Kodydek, 1998) is a three-dimensional (regular) special case of this problem category, in which a given number of identical containers has to be filled with a (strongly heterogeneous) set of items of given weights and values. The total value of the packed items has to be maximised (three-dimensional, rectangular SKP). According to the authors' knowledge, this is the only representative of this problem type discussed in the literature so far.

### 7.7. Multiple Heterogeneous Knapsack Problem

Martello and Toth (1990, pp. 157–187), also see Pisinger (1999) consider the *0-1 Multiple Knapsack Problem*, which is a one-dimensional representative of this problem type (i.e. a one-dimensional MHKP). A strongly heterogeneous set of small items, each of which is characterised by a specific weight and profit, has to be packed into a set of knapsacks of distinct capacities. For each knapsack,

the packed items must not exceed the available capacity, and the total profit of the small items which are packed has to be maximised.

### 7.8. Open Dimension Problem

In the literature, Open Dimension Problems are usually discussed for a single large object. Obviously, problems of this kind are only possible in two and more dimensions.

The (*Two-Dimensional*) *Strip Packing Problem* is an Open Dimension Problem in which a set of two-dimensional small items has to be laid out on a rectangular large object; the width of the large object is fixed, its length is variable and has to be minimised. In case the small items are rectangles (cf. Kröger, 1995; Jacobs, 1996; Hopper and Turton, 2001a; Martello et al., 2003) one may also refer to this problem as the *Rectangular Strip Packing Problem*, or even as the *Orthogonal Rectangular Strip Packing Problem*, if the small rectangles have to be laid out on the large object orthogonally. If the rectangles have to be packed “in levels”, this problem is also called *Level Packing Problem* (cf. Lodi et al., 2004).

In case the small items are non-regularly shaped objects (Oliveira and Ferreira, 1993; Bennell and Dowland, 2001), like in the shoe-manufacturing industry, where they may represent pieces of shoes to be cut from a roll of leather, the problem (i.e. in case of the two-dimensional, irregular ODP, according to the notation introduced here) is also referred to as the *Irregular Strip Packing Problem* (Hopper and Turton, 2001b, p. 257) or the *Nesting Problem* (cf. Oliveira and Ferreira, 1993; Oliveira et al., 2000; Carravilla et al., 2003). With respect to applications in specific areas, the same problem may also be known under different names (e.g., as the *Marker-Making Problem* in the apparel industry; cf. Li and Milenkovic, 1995).

A three-dimensional, rectangular Open Dimension Problem with a single variable dimension (length) occurs in distribution planning, when a given set of cargo (small items) has to be fitted into a container in such a way that the least space in terms of container length is used (Scheithauer, 1991; Miyazawa and Wakabayashi, 1997).

In the *Minimal Enclosure Problem* (Milenkovic and Daniels, 1999), also called *Rectangular Packing Problem* (Hifi and Ouafi, 1998), a set of two-dimensional items has to be laid out such that it can be included in a rectangular (large) object of minimal area. In this case, the extension of the large objects

in both dimensions (width and length) is variable. The small items may be of rectangular (cf. Hifi and Ouafi, 1998), circular (Stoyan and Yaskov, 1998), or irregular shape (Milenkovic and Daniels, 1999).

At least to our knowledge, Open Dimension Problems with more than one large object have not been discussed frequently in the area of C&P. Benati (1997) considers a two-dimensional problem with several large objects of different widths which all have an infinite length. However, we would like to point out that the well-studied (*Classic*) *Multi-processor Scheduling Problem* could be interpreted as a one-dimensional problem of this kind. In this problem, a given set of indivisible jobs (small items) with given processing times (length) has to be allocated to a given number of identical processors (large objects) such that the maximal completion time (also called “schedule length”) is minimised (cf. Heuer, 2004; Brucker, 2004, pp. 107–154; Błażewicz et al., 2001, pp. 137–203). The completion time of a processor is the time necessary to process all the jobs which have been assigned to this processor. In other words, an identical, minimal time-capacity (extension in the variable dimension) has to be assigned to each of the processors that is large enough that all the jobs can be completed. Due to the fact that the Multiprocessor Scheduling Problem and other related problems are hardly ever discussed with respect to C&P problems, we exclude Open Dimension Problems with more than one large object from the following considerations.

### 7.9. Single Stock-Size Cutting Stock Problem

Problems of this type include the *Classic One-Dimensional Cutting Stock Problem* (Gilmore and Gomory, 1963; Wäscher and Gau, 1996), in which standard (or: stock) material of a specific, single length has to be cut down into a weakly heterogeneous set of order lengths (small items). In the *Classic Two-Dimensional Cutting Stock Problem* (Gilmore and Gomory, 1965), a weakly heterogeneous set of order rectangles has to be cut from stock plates of a specific, single size (length and width). In both problems, the number or the value of the necessary large objects (stock lengths or stock plates) has to be minimised.

Examples for a three-dimensional (rectangular) problem of this type include the *Multi-Pallet Loading Problem* (as discussed in Terno et al., 2000) and the *Multi-Container Loading Problem* (Scheithauer, 1999), in which a weakly heterogeneous

assortment of cargo (i.e. a set of boxes) is to be packed on a minimum number of pallets or into a minimum number of containers (also see Bortfeldt, 2000).

### 7.10. Multiple Stock-Size Cutting Stock Problem

Problems of this kind include the natural extensions of the one-dimensional and two-dimensional cutting stock problems to more than one stock size (see Gilmore and Gomory, 1961; Rao, 1976; Dyckhoff, 1981; Scheithauer, 1991; Belov and Scheithauer, 2002 for the one-dimensional and Riehme et al., 1996; Morabito and Arenales, 1996 for the two-dimensional, rectangular case). The One-Dimensional Multiple Stock-Size Cutting Stock Problem has also been considered under the name “*Paper Trim Problem*” in the literature (Golden, 1976, p. 265f.). For a two-dimensional application (i.e. a two-dimensional, rectangular MSSCSP) from furniture manufacturing (see Carnieri et al., 1994).

One of the container-loading problems discussed in Eley 2003 represents a three-dimensional (rectangular) case of this problem type. Both containers and boxes can be grouped into classes. Associated with each container type are specific costs, the total costs of the containers necessary to accommodate all boxes are to be minimised.

### 7.11. Residual Cutting Stock Problem

For a Cutting Stock Problem with a strongly heterogeneous assortment of large objects we have chosen the name “*Residual Cutting Stock Problem*” here, because in practice this case comes about whenever large objects are to be used which represent unused parts of input material (“left-overs”) from previous C&P processes. Under the name of “*Hybrid One-Dimensional Cutting Stock Problem*” a one-dimensional case of this problem type has been introduced in the literature (cf. Gradišar et al., 2002, p. 1212; Gradišar et al., 1999). The authors introduce a one-dimensional cutting problem, in which a weakly heterogeneous assortment of order lengths has to be cut from a set of input lengths which are all different. The supply of input lengths is sufficient to satisfy the demands, and the objective is to minimise the trim loss of the input lengths which are to be used (also see Gradišar et al., 1999, p. 559, case 1). They argue that traditional, pure item-oriented or pure pattern-oriented

solution approaches are not appropriate. Instead, they suggest a new solution approach which is a combination of both. The two-dimensional case is considered in Vanderbeck, 2001. Extensions of this problem type into three or even more dimensions have not been discussed in the literature so far.

### 7.12. Single Bin-Size Bin Packing Problem

The *Classic (One-Dimensional) Bin Packing Problem* is a representative of this problem type. It requires packing a given set of distinct small items of given weights to a minimal number of large objects (bins) of identical size (capacity) such that for each bin the total capacity of the small items does not exceed its capacity (cf. Martello and Toth, 1990; Scholl et al., 1997; Schwerin and Wäscher, 1997). We also note that this problem type has also been named *Vehicle Loading Problem* (cf. Golden, 1976, p. 266) and *Binary Cutting Stock Problem* (cf. Vance et al., 1994) in the literature. The *k-Item Bin Packing Problem* (cf. Babel et al., 2004) additionally requires the assignment of at most  $k$  items to each bin.

What is called the *Two-Dimensional (Orthogonal) Bin Packing Problem* (Lodi et al., 1999, 2002b, p. 379; Lodi et al., 2002a, p. 242; Martello and Vigo, 1998 also refer to this problem type as the *Two-Dimensional Finite Bin Packing Problem* in order to distinguish it from the *Two-Dimensional Strip Packing Problem*, in which the large object has an infinite extension in one dimension) consists of assigning a set of distinct rectangles orthogonally to a minimum number of rectangular bins. According to the suggested typology this is a two-dimensional, rectangular SBSBPP type. George et al. (1995, p. 693) mention the *Cylindrical Bin Packing Problem*. This is a two-dimensional circular SBSBPP, in which the large objects are rectangles and the small items are circles. Real problems of this kind arise in logistics when (a minimum number of) containers are to be loaded with pipes.

In the *Three-Dimensional (Orthogonal) Bin Packing Problem* the items are assumed to be rectangular boxes which are to be fitted orthogonally into a minimal number of rectangular containers of identical size (cf. Lodi et al., 2002c). The *Cube Packing Problem* is a special case of the three-dimensional rectangular Bin Packing Problem, in which all boxes and bins are cubes (Miyazawa and Wakabayashi, 2003).

### 7.13. Multiple Bin-Size Bin Packing Problem

The (*One-Dimensional*) *Variable-Sized Bin Packing Problem* (Chu and La, 2001; Kos and Duhovnik, 2002) is an extension of the Classic One-Dimensional Bin Packing Problem in which several bin types are introduced (i.e. it is a one-dimensional MBSBPP). Each bin type is in unlimited supply and characterised by specific costs and size. All the small items have to be assigned to bins, and the total costs of the used bins have to be minimised (cf. Kang and Park, 2003). A two-dimensional, rectangular case is considered in Tarasova et al., 1997, in which the large objects are in limited supply and where the total area of the material necessary to cut all small items is to be minimised.

### 7.14. Residual Bin Packing Problem

In analogy to the cutting stock problem, for this problem type which is characterised by a set of strongly heterogeneous large objects we have chosen the name “*Residual Bin Packing Problem*”.

Chen et al. (1995) consider a three-dimensional container loading problem, in which a weakly heterogeneous consignment of boxes has to be packed into containers available in different sizes. In one of the discussed situations, the available container space is not sufficient to accommodate all boxes. Therefore, a selection of boxes has to be determined that maximises the volume utilisation, or, alternatively, the value of the packed boxes (three-dimensional, rectangular RBPP).

## 8. Categorisation of recent literature (1995–2004)

In order to demonstrate the practicability and usefulness of the suggested typology, the recent C&P literature has been reviewed and categorised according to the scheme introduced above. We concentrated on papers which are publicly available and have been published in English in international journals, edited volumes, or conference proceedings during the decade between 1995 and 2004. Monographs and working papers have not been considered in our investigation.

In order to exclude publications from the periphery of C&P from our analysis, which are only of marginal interest to researchers and practitioners in the field, we restricted ourselves to papers related to C&P problems “in a narrow sense” (Dyckhoff, 1990, p. 148) in the first place. Papers on “abstract”

C&P problems (Dyckhoff, 1990, p. 148) have only been considered as far as these problems (like those of the knapsack type) have been introduced in the suggested typology, i.e. we refrained from including other papers from this category not addressing C&P directly, like papers on flow-line balancing (Talbot et al., 1986; Scholl, 1999), multi-processor scheduling (Brucker, 2004, pp. 107–154; Błażewicz et al., 2001, pp. 137–203), or capital budgeting (Lorie and Savage, 1955).

Furthermore, only papers dealing with C&P problems in the sense of refined problem types have been included in our analysis. Articles dealing with problem extensions and problem variants were not taken into account for. We note again that we have excluded papers of this kind only in order to keep the number of papers to be considered to a manageable size and to find a definition of a paper cluster that is of interest to researchers and practitioners in the field of C&P. Focussing our investigation in this particular way by no means limits the usefulness and the value of the suggested typology. In fact, also the problems discussed in papers which have been excluded here can be categorised according to our typology.

Finally we remark that only such papers have been categorised which strictly satisfy the above-given definition of C&P problems and subsequent specifications. That means that papers, e.g. dealing with divisible small items and/or large objects (Kang and Park, 2003), or allowing the “overpacking” of bins (Coffman and Lueker, 2001) do not appear in our analysis.

As far as for December 2005, 413 papers have been identified containing material relevant in the above-described sense. These papers are listed at [http://www.uni-magdeburg.de/mansci/rm/cp\\_typology](http://www.uni-magdeburg.de/mansci/rm/cp_typology), together with the corresponding problem types to which they have been assigned. Table 1 shows what problem types and number of dimensions are dealt with in these papers. The total numbers given in this and the following tables is larger

Table 1  
Number of problem-relevant dimensions and assignment types of problems dealt with in the literature

Kind of assignment	1D	2D		3D	Total
		regular	irregular		
Input minimisation	108	79	52	24	263
Output maximisation	64	71	12	35	182
Total	172	150	64	59	445

than 413 because in some papers more than one problem type is addressed. Consequently, those papers had to be counted more than once.

Research on input minimisation problems (dealt with in 263 papers, 59%) clearly dominates research on output maximisation problems (182 papers, 41%). One-dimensional and two-dimensional prob-

lems (172 papers, 39%, and 214 papers, 48%) are considered significantly more often than three-dimensional ones (59 papers, 13%).

Figs. 12 and 13 give a more detailed analysis of how the papers are distributed over the different problem categories. Among research on output maximisation problems, papers on problems with a sin-

characteristics of the large objects		assortment of the small items		
		identical	weakly heterogeneous	strongly heterogeneous
all dimensions fixed	one large object	IIPP 23	SLOPP 56	SKP 86
	identical	X	MILOPP 1	MIKP 6
	heterogeneous		MHLOPP 4	MHKP 6

Fig. 12. Distribution of publications: output maximisation.

characteristics of large objects		assortment of small items	
		weakly heterogeneous	strongly heterogeneous
all dimensions fixed	identical	SSSCSP 38	SBSBPP 89
	weakly heterogeneous	MSSCSP 18	MBSBPP 4
	strongly heterogeneous	RCSP 10	RBPP 2
one large object variable dimension(s)		ODP 102	

Fig. 13. Distribution of publications: input minimisation.

gle large object are prevailing (165 papers, 37% of all papers and 91% of those belonging to the output-maximisation type). With respect to input minimisation, most of the publications deal with problems in which the assortment of large objects either consists of identical large objects of a single given size (127 papers, 29% of all papers, and 48% of those belonging to the input-minimisation type), or, alternatively, of a single large object with one variable dimension (102 papers, 23 and 39%).

As becomes further evident, published research concentrates on five problem types, namely on the ODP (102 papers, 23%), SBSBPP (89 papers, 20%), SKP (86 papers, 19%), SLOPP (56 papers, 13%) and the SSSCSP (38 papers, 9%). Papers on these five problem types account for 371 out of 445 publications (83%). In Table 2 the numbers of papers which consider these problem types are further differentiated with respect to the number of problem-relevant dimensions.

Table 2 reveals that research on C&P still is rather traditionally oriented. It stresses areas which include clearly-defined (“classic”) standard problems, well-studied for three decades or an even longer period of time, such as the one-dimensional SBSBPP (including the Classic Bin Packing Problem), the two-dimensional ODP (including the Regular and the Irregular Strip-Packing Problem), the one-dimensional SSSCSP (including the Classic Cutting Stock Problem), the one-dimensional SKP (including the Classic Knapsack Problem), and the two-dimensional SLOPP (including the Distributor’s Pallet Loading Problem). As far as research departs from these traditional areas, it is devoted to straightforward extensions of these standard problems into a higher number of dimensions, e.g., to the two-dimensional Bin Packing Problem, the two-dimensional Knapsack Problem, etc. Other kinds of problem extensions (e.g., considering differ-

ent assortments of large objects and small items) which are probably more relevant to the solution of real-world C&P problems can only be found far less commonly in the literature, even though a few, more recent papers considering a heterogeneous assortment of large objects (51) seem to indicate that the preferences of researchers might be changing.

We finally note (cf. Fig. 12) that the IIPP which has been extensively studied in the 1980s is only represented in 23 papers. This seems to indicate that the central standard problem of this type, the classic Manufacturer’s Pallet Loading Problem, has been solved satisfactorily and that research has shifted to more complex problem situations recently, taking care, in particular, of a (weakly) heterogeneous assortment of small items (boxes to be packed).

## 9. Outlook

The suggested typology should be sufficient for an initial, brief orientation in a particular area of C&P. To practitioners and researchers confronted with particular C&P problems Fig. 2 points out the relevant problem parameters, allowing them to assign their problems to the relevant problem categories. An extensive database of C&P publications (also including those concerning problem extensions and problem variants), in which papers are classified and organised in accordance with the suggested typology, is available at the ESICUP webpage (<http://www.apdio.pt/sicup/>). It provides direct access to the literature of each specific problem type.

Fig. 2 also outlines the area of future work. In order to structure the refined problem types further, it will be necessary to compile, analyse and group the respective additional constraints. Furthermore, problem extensions and problem variants and the corresponding standard problems have to be investigated in greater detail.

## Acknowledgements

The basic ideas of this typology have been presented twice at meetings of the EURO Working Group “ESICUP”, namely at Lutherstadt Wittenberg in 2004 and at Southampton in 2005. The authors are thankful to the participants for their various critical comments which helped to elaborate the basic structure of the typology more clearly. José Fernando Oliveira deserves particular thanks

Table 2  
Number of papers on selected problem types, differentiated according to the number of problem-relevant dimensions

Problem types	1D	2D	2D	3D	Total
		regular	irregular		
ODP	–	46	49	7	102
SBSBPP	61	17	2	9	89
SKP	49	18	7	12	86
SLOPP	4	32	1	19	56
SSSCSP	29	2	1	6	38
Other	29	35	4	6	74
Total	172	150	64	59	445

for his continuous encouragement and availability for discussions throughout the preparation of this paper.

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