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Abstract

In manual order picking systems, order pickers walk or ride through a distribution warehouse in order to collect items requested by (internal or external) customers. In order to perform these operations efficiently, it is usually required that customer orders are combined into (more substantial) picking orders of limited size. The Order Batching Problem considered in this paper deals with the question of how a given set of customer orders should be combined such that the total length of all tours necessary to collect all items is minimized. For the solution of this problem the authors suggest two approaches based on the tabu search principle. The first one is a straightforward classic Tabu Search algorithm (TS), the second one is the Attribute-Based Hill Climber (ABHC). In a series of extensive numerical experiments, the newly developed approaches are benchmarked against different solution methods from literature. It is demonstrated that the proposed methods are superior to existing methods and provide solutions which may allow for operating distribution warehouses significantly more efficiently.

Keywords: Logistics, Warehouse Management, Order Batching, Tabu Search, Attribute-Based Hill Climber

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1 Introduction

Order picking is a warehouse function dealing with the retrieval of articles (items) from their storage location in order to satisfy a given demand specified by (internal or external) customer orders (Petersen and Schmenner, 1999). Order picking arises because incoming articles are received and stored in (large-volume) unit loads while customers order small volumes (less-than-unit loads) of different articles. On the one hand, underperformance in order picking may result in an unsatisfactory customer service (long processing and delivery times, incorrect shipments); on the other, it may lead to high costs (labor cost, cost of additional and/or emergency shipments), since more than 50% of the total operative costs of a warehouse are related to order picking (Frazelle, 2002). Both aspects may have a negative impact on the competitiveness of the warehouse.

Like many other repetitive material-handling activities, order picking is still a function which involves the employment of human operators at a large scale. Even though various attempts have been made to automate the picking process, manual order picking systems are still prevalent in practice. Such systems can be differentiated into two categories: picker-to-parts systems, in which order pickers walk or ride through the warehouse and collect the requested items, and parts-to-picker systems, in which automated storage and retrieval systems deliver the items to stationary order pickers (Wäscher, 2004). With respect to systems of the first kind, three planning problems can be distinguished on the operative level (Caron et al., 1998): the assignment of articles to storage locations (article location), the grouping of customer orders into picking orders (order batching), and the routing of order pickers through the warehouse (picker routing).

This paper deals with the second activity, which has proven to be pivotal for the efficiency of warehouse operations (de Koster et al., 1999).

Henn et al. (2009) have shown that the application of a local search-based metaheuristic, namely Iterated Local Search, to the Order Batching Problem can improve the warehouse efficiency substantially. In their experiments, however, the application of the proposed algorithm turned out to be very time-consuming, and, in particular, not suitable for picking environments where solutions have to be generated quickly or even in real time. This article focuses on two other types of local search-based approaches, which have demonstrated to provide high-quality solutions within marginal computing times for related combinatorial optimization problems. The first one is a straightforward application of Tabu Search (TS); the second one is the Attribute-Based Hill Climber (ABHC), which is also based on a tabu search principle, but does only require to specify a small number of parameters.

The remainder of the paper is organized as follows: In Section 2 the Order Batching Problem (OBP) will be defined. Section 3 contains a literature review of solution approaches for the OBP. In the subsequent section the straightforward TS implementation is described. In Section 5 it is shown how the general principle of ABHC can be applied to the OBP. Extensive numerical experiments have been carried out in order to evaluate the performance of the metaheuristics. Section 6 is dedicated to the design of these experiments (including the description of warehouse parameters, algorithm parameters, and problem classes). The performance of TS and ABHC is analyzed for different configurations in Section 7 and Section 8, respectively. A comparative analysis of the proposed method against benchmark heuristics and optimal solutions is presented in Section 9. The article concludes with a summary on the main contributions of the work and an outlook on further research opportunities.

2 Order Batching Problem

2.1 Problem Description

A customer order consists of a (non-empty) set of order lines, where each order line consists of a particular article and the corresponding requested quantity. A pick list contains the order lines which should be processed together and guides the order picker through the warehouse. This list may contain the order lines of a single customer order (pick-by-order) or of a combination of customer orders (pick-by-batch). In practice, the sequence in which the items are to be picked and the corresponding route of the order picker (who starts at the depot, proceeds to the respective storage locations, and returns to the depot) are usually determined by means of a so-called routing strategy. Since order pickers seem to accept only straightforward and non-confusing routing schemes, routes provided by the S-Shape- or Largest Gap-Heuristics are prevalent in practice (de Koster et al., 1999). The character of these heuristics is exemplified in Figure 2.1. The black rectangles symbolize the locations of items to be picked on the respective routes.

Solutions provided by the *S-Shape-Heuristic* are characterized as follows: the order picker enters an aisle if at least one requested item is located in that aisle and traverses the aisle completely. Afterwards, the order picker proceeds to the next aisle which contains a requested item. An exception could be the last aisle: if the order picker is positioned in the front cross-aisle, he/she would pick the items of the last aisle and return to the depot along the front aisle. The *Largest Gap-Heuristic* gives a solution in which the order picker entirely traverses the rightmost and leftmost aisle containing an item to be picked. All the other aisles are entered from the front and back aisle in such a way that the non-traversed distance between two adjacent locations of items to be picked in the aisle is maximal.

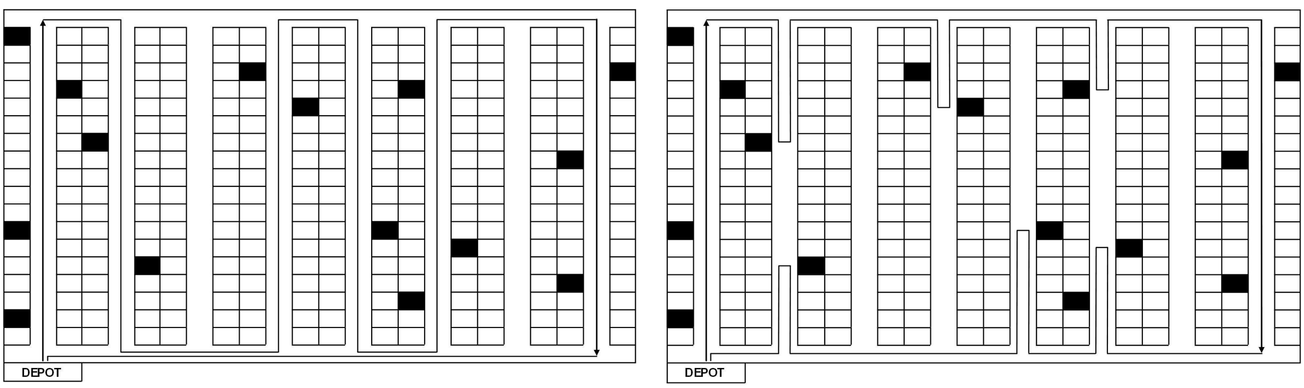

Figure 2.1: Example of S-Shape (left) and Largest Gap (right) in a single-block warehouse

For the collection of the requested items, order pickers are usually taking advantage of a picking device (e.g. a cart or a roll pallet). Customer orders can be combined into picking orders until the capacity of the device is exhausted. Splitting of customer orders is usually not permitted since it would result in an additional, not acceptable sorting effort.
With respect to the availability of information of customer orders, order batching can be distinguished into off-line batching and on-line batching (Yu and de Koster, 2009). The off-line case, which is covered in this paper, considers an order picking environment where all customer orders are known in advance. In the on-line case customer orders arrive at different points in time.
Due to the high amount of time-consuming manual operations, the minimization of picking times is of great importance for controlling the picking process efficiently. The time spent by order pickers to collect the items of all customer orders (total picking time) consist of the setup times for the routes, the travel times that are needed to travel to, from, and between the locations of items to be picked, the search times for the identification of the items, and the times needed for picking the items. Among these components, the travel time is of outstanding importance, since it consumes the major proportion of the total order picking time (at least 50% of the order picker's time), while the other components can either be assumed as constant (search times and pick times) or as neglectable (setup times) (Tompkins et al., 2003). Furthermore, given that the pickers' travel velocity is constant, the minimization of the total travel time is equivalent to the miminization of the total length of all picker tours (Jarvis and McDowell, 1991).
The Order Batching Problem can then be summarized and defined as follows: How can a given set of customer orders, with given storage locations, given routing strategy and given capacity of the picking devices, be grouped (batched) into picking orders such that the total length of all necessary picking tours (total tour length) is minimized (Wäscher, 2004).

2.2 Model Formulation

According to Gademann and van de Velde (2005), a mathematical formulation of the problem can be given as follows: Let $J = \{1, \ldots, n\}$ be the set of customer orders, C the capacity of the picking device, and c_j the capacity required for order j ($j \in J$). Furthermore, let each batch of customer orders be described by a vector $a_i = (a_{i1}, ..., a_{in})$ with binary entries a_{ij} stating whether an order j is included in a batch i ($a_{ij} = 1$) or not ($a_{ij} = 0$). The set I of all feasible batches is characterized by the fact that the capacity of the picking device is not violated, i.e. the following property holds:

$$\sum_{j \in J} c_j \cdot a_{ij} \leq C, \ \forall i \in I. \tag{2.1}$$

Let x_i ($i \in I$) be a binary decision variable, which describes if a batch i is chosen ($x_i = 1$) or not ($x_i = 0$), and let d_i further represent the length of the picking tour in which all orders of a batch

i are collected, then the following optimization model can be formulated for the OBP:

$$\min \sum_{i\in I} d_i \cdot x_i \tag{2.2}$$

$$\text{s.t.} \sum_{i\in I} a_{ij} \cdot x_i = 1, \ \forall j \in J; \tag{2.3}$$

$$x_i \in \{0,1\}, \ \forall i \in I. \tag{2.4}$$

The sets of constraints (2.3) and (2.4) ensure that a set of batches is chosen in a way that each customer order is included in exactly one of the chosen batches. The number of possible batches and, therefore, the number of binary variables grows exponentially with the number of orders.
The OBP as described above is known to be $\mathcal{NP}$-hard (in the strong sense) if the number of orders per batch is greater than two (Gademann and van de Velde, 2005).

3 Literature Review

So far, only few approaches have been described in the literature which solve the OBP to or close to optimality. Gademann and van de Velde (2005) describe a mixed integer programming model and a branch-and-price algorithm with column generation that was able to provide optimal solutions for small instances (up to 32 orders). For the case of S-Shape-Routing Bozer and Kile (2008) present a mixed integer programming approach that generates near optimal solutions for small (up to 25) order sizes.
For larger problems – as they usually occur in practice – the use of heuristics is essential. Heuristics for the OBP can be distinguished into four groups: priority rule-based algorithms, seed algorithms, savings algorithms, and metaheuristics. The first three groups contain approaches of the constructive type (Wäscher, 2004), whereas metaheuristic approaches are mainly of an improvement type.
In *priority rule-based algorithms* the customer orders are first ranked according to a priority value and then assigned to batches following this ranking (Gibson and Sharp, 1992). The probably best known and most straightforward way consists of the application of the First-Come-First-Served rule (FCFS). Other priority rules include space-filling curves (Gibson and Sharp, 1992; Pan and Liu, 1995). The assignment of customer orders to batches can either be done sequentially (Next-Fit-Rule) or simultaneously (First-Fit-Rule, Best-Fit-Rule) (Wäscher, 2004).
The second group, introduced by Elsayed (1981) and Elsayed and Stern (1983), is formed by *seed algorithms*, which generate batches sequentially. They select one customer order as an initial order for a batch. Additional customer orders are assigned to that batch according to an order-congruency rule. An overview of the various seed selection and order-congruency rules is given by de Koster et al. (1999), Ho and Tseng (2006) and Ho et al. (2008).
Savings algorithms are based on the Clarke-and-Wright-Algorithm for the Vehicle Routing Problem (Clarke and Wright, 1964) and have been adapted in several ways for the OBP. In its initial version for the OBP (C&W(i)), for each combination of customer orders i and j the savings sav_{ij}

are computed which can be obtained by collecting the items of the two customer orders on one (large) tour instead of collecting them in two separate tours. Starting with the pair of orders with the highest savings, the pairs are considered for being assigned to a batch in a non-ascending order. This may lead to three situations: in case that none of the two orders has been assigned, a new batch will be opened for them; if one of the orders has already been assigned, the other one is added to the batch if the remaining capacity is sufficient; in case that not enough capacity is available or that both orders have already been assigned, the next pair of orders will be considered. All orders left unallocated at the end of the process will be assigned to an individual batch each. The algorithm can be improved by recalculating the savings each time orders have been allocated to a batch (C&W(ii)) (Elsayed and Unal, 1989).
Comprehensive numerical experiments (de Koster et al., 1999) have shown that among the constructive heuristics either seed algorithms or savings algorithms provide the best solutions with respect to the total tour length, dependent on the warehouse layout and on customer order characteristics.
Finally, the last group of solution approaches to the OBP is formed by *metaheuristics*. Hsu et al. (2005) have suggested a genetic algorithm for the OBP. Their approach includes an aisle-metric for the determination of the tour lengths and is, therefore, limited to S-Shape-Routing, only. Tsai et al. (2008) describe an integrated approach, in which both batching and routing problems are solved by means of a genetic algorithm. Due to these specific conditions, both approaches will not be considered here any further. Gademann and van de Velde (2005) propose an iterated improvement algorithm that incorporates a 2-opt neighborhood. Henn et al. (2009) apply two methods for solving the OBP, namely Iterated Local Search and a Rank-Based Ant System. By means of numerical experiments the authors demonstrate that both approaches outperform the constructive algorithms, yet at the expense of long computing times.
Finally, Chen and Wu (2005) describe an order batching approach based on data mining and integer programming. In this approach, at first similarities of customer orders are determined by means of an association rule; then a 0-1 integer programming approach is applied in order to cluster the orders into batches.

4 Tabu Search

4.1 General Principle

In combinatorial optimization, a solution S has to be found in the set of all feasible solutions $\mathcal{F}$ which has a minimal (or maximal) value for an objective function d. A minimization problem can, therefore, be described as

$$\min\{d(S)|S \in \mathcal{F}\}. \tag{4.1}$$

A simple and widely-used method to solve combinatorial optimization problems is local search (LS): For a solution S ($S \in \mathcal{F}$) the subset $\mathcal{N}(S)$ of $\mathcal{F}$ is called *neighborhood of* S in which each solution $S' \in \mathcal{N}(S)$ can be obtained by applying a single local transformation ("move") to S. Local search generates a sequence of solutions $S_0, S_1, S_2, \ldots$, where each member of the sequence is an element of the neighborhood of its predecessor. Each element possesses a smaller objective function value than the previous one. The disadvantage of local search is that the sequence terminates at a local optimum, which – for large problem sizes – does not necessarily represent a global optimum, and the corresponding objective function value may differ significantly from to the optimal one. Therefore, different approaches based on the LS principle have been designed in order to overcome such local optima by allowing also deteriorations of the objective function value during the search.

Tabu Search – as a specific local search-based method – was developed by Glover (1986) and aims at simulating human memory processes. This memory process is implemented by a tabu list which records moves applied in previous iterations. The application of these moves is forbidden ("tabu") for a particular number of iterations in order to avoid cycling and to diversify the search. In each iteration Tabu Search considers only those elements of the neighborhood as new incumbent solutions which can be obtained by a non-tabu move. After having accepted a new solution, the tabu list is updated. The algorithm stops when a termination condition is met. Algorithm 4.1 summarizes the general principle of Tabu Search, to which a variety of extensions and features can be added (Glover and Laguna, 1997).

Algorithm 4.1 Tabu Search: General Principle

Input: initial solution S_0, tabu list T;

```
S* := S_0;
repeat
  generate N(S_i);
  delete solutions from N(S_i) which were obtained by a tabu move;
  select solution from N(S_i) as next solution S_{i+1};
  update T according to S_i, S_{i+1} and i;
  i := i + 1;
until termination condition is met;
```

4.2 Specifications for the OBP

For the generation of the initial solution we suggest two options, namely generation according to the *FCFS-rule* (FCFS in this context means that costumer orders are assigned to batches according to their index) and generation by application of C&W(ii). Whereas a solution (defined as a set of batches) generated by FCFS is a totally random solution, solutions obtained by C&W(ii) may already be identical to or close to local minima.

For the neighborhood structure of a solution S we consider three different options. The first one, $\mathcal{N}_{\text{swap}}(S)$, is the set of solutions which can be obtained from S by interchanging two orders from different batches (*swap*). The second one, $\mathcal{N}_{\text{shift}}(S)$, is the set of solutions which can be generated by assigning one order to a different batch (*shift*). The third neighborhood structure is defined by a combination of swaps and shifts, i.e. the set of solutions is given by $\mathcal{N}_{\text{com}}(S) := \mathcal{N}_{\text{swap}}(S) \cup \mathcal{N}_{\text{shift}}(S)$. All three neighborhood structures ($\mathcal{N}_{\text{swap}}$, $\mathcal{N}_{\text{shift}}$, $\mathcal{N}_{\text{com}}$) have a respective size of $O(n^2)$, where n is the total number of customer orders. Moves are recorded on a tabu list in such a way that after having swapped order i of batch j with order i' of batch j', this swap and the corresponding re-exchange cannot be applied for a particular number of iterations. For the shift of order i from batch j to batch j' this transformation and its inversion are tabu. The number of iterations for which a move remains on the tabu list (tabu tenure) is determined dynamically and randomly chosen from the set $\{1.5n, \ldots, 2n\}$.

Two options are suggested for the exploration of the neighborhood of the incumbent solution: In the *best improvement-strategy* (BI) the complete neighborhood is generated and the non-tabu solution with the smallest objective function value is selected as next incumbent solution. In order to save computing time, the *aspiration plus-criterion* (AP) is used to explore a limited subset of the neighborhood of the incumbent solution. The algorithm generates solutions of the neighborhood until a (non-tabu) solution is found with a total tour length which is at most 5% larger than the objective function value of the incumbent solution. Afterwards, n further non-tabu solutions are generated. In addition, we enforce that at least $3n$ and at most $5n$ solutions are considered. From these solutions we select the one with the smallest total tour length as the next incumbent solution.

If the neighborhood of a solution contains only solutions which can be obtained by a tabu move, then those moves are deleted from the tabu list which have the smallest remaining tabu tenure (*aspiration by default*). If a tabu solution of the neighborhood has the smallest total tour length found so far, it would be accepted as new incumbent solution, ignoring its tabu status (*aspiration by objective*). The algorithm terminates after having executed $30n$ iterations without any improvement of the best solution found.

We note that each proposed configuration of TS can be described by the triple TS(./././), where the first argument describes the generation of the initial solution (FCFS or C&W(ii)), the second the neighborhood structure ($\mathcal{N}_{\text{swap}}$, $\mathcal{N}_{\text{shift}}$, or $\mathcal{N}_{\text{com}}$) and the third the way according to which the neighborhood (BI or AP) is explored.

5 Attribute-Based Hill Climber

5.1 General Principle

The Attribute-Based Hill Climber is an almost parameter-free heuristic based on a simple tabu search principle. It was developed by Whittley and Smith (2004), who applied it successfully to the Traveling Salesman and to the Quadratic Assignment Problem. Derigs and Kaiser (2007) proposed an application of ABHC to the Vehicle Routing Problem and showed that the quality of the generated solutions is competitive with that of solutions generated by other state-of-the-art heuristics. Furthermore, Derigs and Reuter (2009) demonstrated that ABHC was able to find

high quality solutions for the Open Vehicle Routing Problem.
The main principle of ABHC can be described in the following way: For each problem a set of attributes $\mathbf{A} = \{A_1, \ldots, A_q\}$ is introduced. An attribute can be any specific solution feature. In order to describe whether a solution 'contains' an attribute A_k ($k \in \{1, \ldots, q\}$) or not, a Boolean function $a_k : \mathcal{F} \mapsto \{\text{'true', 'false'}\}$ can be introduced where

$$a_k(S) = \begin{cases} \text{true} & \text{if } S \text{ contains attribute } A_k, \\ \text{false} & \text{else.} \end{cases} \tag{5.1}$$

For each solution S ($S \in \mathcal{F}$) and each attribute A_k ($k \in \{1, \ldots, q\}$), $d_k(S)$ can be defined as

$$d_k(S) = \begin{cases} d(S) & \text{if } a_k(S) = \text{'true'}, \\ \infty & \text{else.} \end{cases} \tag{5.2}$$

ABHC performs a local search, in which solutions $S_0, S_1, S_2, \ldots$ are visited. Let S_i be the solution visited in iteration i, then for each attribute k the best objective function value $\text{best}(i, k)$ for a solution containing this attribute is recorded,

$$\text{best}(i, k) := \begin{cases} d_k(S_0) & \text{for } i = 0, \\ \min\{\text{best}(i-1, k), d_k(S_i)\} & \text{for } i \geq 1. \end{cases} \tag{5.3}$$

A solution can be accepted if and only if it possesses for at least one attribute that 'contains' it the smallest objective function value found so far. In each iteration i the set of acceptable solutions can be defined as

$$\mathcal{F}_i := \{S | \exists k \in \{1, \ldots, q\} \text{ with } a_k(S) = \text{true and } d(S) < \text{best}(i, k)\}. \tag{5.4}$$

Summarize, the algorithm starts with an initial solution S_0 and searches for a new incumbent solution in each iteration i. For this solution only neighbors of the incumbent solution (elements of $N(S_i)$) are considered which possess for at least one containing attribute the best found objective function value (elements of $\mathcal{F}_i$). Among this set $\mathcal{N}(S_i) \cap \mathcal{F}_i$ an element with the smallest objective function value is chosen as new incumbent solution S_{i+1}, and the values $\text{best}(i, k)$ ($\forall k \in \{1, \ldots, q\}$) are updated. The algorithm stops if the current neighborhood contains no solution that represents a best solution for at least one containing attribute, i.e. if $\mathcal{N}(S_i) \cap \mathcal{F}_i = \emptyset$. Since $\mathcal{F}_i$ is finite, ABHC stops after a finite number of iterations. Algorithm 5.1 summarizes the general principle of ABHC.

5.2 Specifications for the OBP

The advantage of ABHC can be seen in the fact that only three general design decisions have to be taken: the choice of the initial solution, the neighborhood structure, and the set of attributes. Each decision influences the efficiency and complexity of the heuristic. Determination of further parameter settings, which have to be derived in a probably lengthy procedure, is not required.

Algorithm 5.1 Attribute-Based Hill Climber: General Principle

Input: initial solution S_0

$S^\star := S_0$;
determine best$(0, k)$;
while $N(S_i) \cap \mathcal{F}_i \neq \emptyset$ **do**
  $S_{i+1} := \arg\min\{d(S) | S \in N(S_i) \cap \mathcal{F}_i\}$;
  $i := i + 1$;
  determine best(i, k);
  update best solution found so far;
end while

For the initial solution and the neighborhood structure we suggest the same options as for TS, namely FCFS and C&W(ii), and $\mathcal{N}_{\text{swap}}$, $\mathcal{N}_{\text{shift}}$, $\mathcal{N}_{\text{com}}$, respectively (cf. Section 4).
With respect to the attributes the following two sets are used: The attribute set $\mathbf{A}_{\text{o, o}} := \{(i_1, i_2)|1 \leq i_1, i_2 \leq n\}$ describes the solution by pairs (i_1, i_2) of customer orders i_1, i_2 which are assigned to the same batch. The second set of attributes captures the assignment of customer orders to batches, i.e. $\mathbf{A}^{\text{lex}}_{\text{b, o}} := \{(j, i)|1 \leq j \leq m, 1 \leq i \leq n\}$. Making use of this set, an arbitrary ordering of the batches would result in identical solutions which can be described by a variety of different attributes. We consider an example where customer orders i_1, i_2 are assigned to batch j_1 and orders i_3, i_4 are assigned to batch j_2. This solution can be described by the attributes $(j_1, i_1), (j_1, i_2), (j_2, i_3), (j_2, i_4)$. Another solution in which orders i_1 and i_2 are assigned to batch j_2 and i_3 and i_4 to batch j_1 is identical to the first solution but results in a different attribute set and would, therefore, be interpreted in ABHC as a different solution. In order to avoid that ABHC tends to explore identical solutions for a larger number of iterations, the batches are ordered according to the smallest index number of the customer orders in the batch.
Each proposed configuration of ABHC will, again, be abbreviated by the triple ABHC(./././), where the first argument describes the initial solution (FCFS or C&W(ii)), the second the neighborhood structure ($\mathcal{N}_{\text{swap}}$, $\mathcal{N}_{\text{shift}}$ or $\mathcal{N}_{\text{com}}$) and the third the attribute set ($\mathbf{A}_{\text{o, o}}$ or $\mathbf{A}^{\text{lex}}_{\text{b, o}}$).

6 Design of the Computational Experiments

6.1 Warehouse Parameters

In order to evaluate the performance of the proposed heuristics numerically, we assume a single-block warehouse with two cross aisles, one in the front and one in the back of the picking area. All aisles are vertically orientated and the depot is located in front of the leftmost aisle. This type of layout is depicted in Figure 2.1 and is frequently used in literature (de Koster et al., 1999; Petersen and Schmenner, 1999; Henn et al., 2009).
The picking area consists of 900 storage locations and we assume that a different article has been assigned to each location. The storage locations are partitioned in 10 (picking) aisles with 90

storage locations each (45 on both sides of each aisle). The aisles are numbered from 1 to 10, where aisle no. 1 is the leftmost aisle, and aisle no. 10 the rightmost aisle. Within an aisle, two-sided picking is assumed, i.e. being positioned in the center of an aisle, the order picker can pick items – without additional movements – from storage locations on the right as well as from those on the left. The length of each cell amounts to one length unit (LU). When picking an article, the order picker is positioned in the front middle of the storage location. Whenever the order picker leaves an aisle, he/she has to move one LU in vertical direction from the first storage location, or from the last storage location respectively, in order to reach the cross aisle. In order to change over into the next aisle, the order picker has to move 5 LU in horizontal direction, i.e. the center-to-center distance between two aisles amounts to 5 LU. The depot is 1.5 LU away from the first storage location of the leftmost aisle, i.e. the distance between cross aisle and depot amounts to 0.5 LU. A class-based storage assignment of articles to storage locations is assumed, i.e. the articles are grouped into three classes A, B and C by their demand frequencies, where A contains articles with high, B with medium and C with low demand frequency. 52 percent of the demand belongs for articles in class A, 36 percent for articles in B and 12 percent for articles in C. Articles of class A are stored in aisle no. 1, articles of class B in aisles no. 2 to 4, and articles of class C in the remaining six aisles. Within a class, articles are located randomly.

6.2 Problem Classes

In our numerical experiments, we consider problems of four different sizes, namely instances with 40, 60, 80, 100 customer orders. The number of items per order is uniformly distributed in $\{5, \ldots, 25\}$. The capacity of the picking device (defined by the maximal quantity of items that can be assigned to a batch) has been fixed to 30, 45, 60 and 75 items, chosen in a way that a batch consists of 2 to 5 customer orders on average. In combination with the two routing strategies (S-Shape, Largest Gap) this gives rise to 40 problem classes, and for each of these classes 40 instances were generated. The instances for the problem classes with 40 and 60 customer orders are identical with those from Henn et al. (2009).

6.3 Upper Bound

Application of the savings algorithm C&W(ii) represents a straightforward way to generate solutions to the Order Batching Problem; it provides an upper bound which has been selected as a baseline, here. The solution quality of all other batching strategies is measured in relation to that of C&W(ii).

6.4 Implementation and Hardware

The computations for all 1,280 instances have been carried out on a desktop PC with a 2.21 GHz Pentium processor with 2.0 GB RAM. The algorithms have been encoded in C++ using the DEV Compiler Version 4.9.9.2.

7 Configuration Pretests for TS

7.1 S-Shape-Routing

Solution Quality

On average, the best results for TS are provided by a configuration TS*(SSR) which is characterized by initial solutions generated by means of C&W(ii), the neighborhood structure $\mathcal{N}_{\text{com}}$, and the aspiration plus criterion, i.e. TS*(SSR):= TS(C&W(ii)/$\mathcal{N}_{\text{com}}$/AP). Improvements as compared to C&W(ii) amount to 4.2% on average, and range from 2.3% (number of customer orders $n = 40$; capacity of the picking device $C = 30$) to 6.2% (40; 60). Table 10.1 in the appendix depicts the average improvement of the total tour length for each problem class for all TS variants and S-Shape-Routing. The worst results can be observed for TS(FCFS/$\mathcal{N}_{\text{swap}}$/BI) where the average total tour length is 4.4% longer than the one provided by C&W(ii).
What concerns the impact of the initial solution on the solution quality, the TS(C&W(ii)/./.) variants outperform the corresponding TS(FCFS/./.) variants. TS(C&W(ii)/$\mathcal{N}_{\text{shift}}$/./) offers average improvements up to 3.2 percentage points compared to TS(FCFS/$\mathcal{N}_{\text{shift}}$/./), and TS(C&W(ii)/ $\mathcal{N}_{\text{swap}}$/.) up to 7.2 compared to TS(FCFS/$\mathcal{N}_{\text{swap}}$/.), while the difference between the results provided by TS(C&W(ii)/$\mathcal{N}_{\text{com}}$/./) and TS(FCFS/$\mathcal{N}_{\text{com}}$/./) are smaller than 0.7 percentage points. It should be noted that FCFS provides solutions with a larger number of batches than C&W(ii) does. The swap-operator does not allow for reducing the number of batches, which explains the unsatisfactionary performance of TS(FCFS/$\mathcal{N}_{\text{swap}}$/.).
Application of TS(./$\mathcal{N}_{\text{comb}}$/.) always achieves significantly better results than application of the two other neighborhood structures. For an increasing capacity of the picking device the improvements obtained by $\mathcal{N}_{\text{shift}}$ decrease, whereas the improvements obtained by $\mathcal{N}_{\text{swap}}$ increase. Therefore, a rank order between TS(./$\mathcal{N}_{\text{shift}}$/.) and TS(././$\mathcal{N}_{\text{swap}}$) cannot be identified.
Analyzing the size of the explored neighborhood, a small advantage (up to 0.4 percentage points on average) can be observed for the application of TS(././AP).

Computing Times

Computing times for instances with 20 or 40 customer orders amount to less than 5 seconds and can be considered as neglectable. For the remaining problem classes the computing times for TS*(SSR) range from 6 seconds (60; 30) up to 42 seconds (100; 75). For a detailed view of the average computing times per instance required by all proposed variants of TS it is referred to Table 10.2 in the appendix. In general, increasing values of the two parameters n and C result in increasing computing times.
The computing times observed for TS(C&W(ii)/./.) are smaller than the corresponding ones for TS(FCFS/./.). The largest differences occur between TS(C&W(ii)/$\mathcal{N}_{\text{com}}$/BI) and TS(FCFS/$\mathcal{N}_{\text{com}}$ /BI), where - on average – the first variant consumes only 60% of the computing time of the latter one.
Even though all neighborhood solutions obtained by the swap-operator are also included in

$\mathcal{N}_{\text{com}}$, the computing times for TS(./$\mathcal{N}_{\text{com}}$/.) are nearly identical or even smaller than those for TS(./$\mathcal{N}_{\text{swap}}$/.). The computing times of TS(./$\mathcal{N}_{\text{shift}}$/.) are significantly smaller than those for TS(./$\mathcal{N}_{\text{com}}$/.).
The computing times for those combinations which make use of AP are significantly smaller (cf. problem class (100; 75)) than for those using BI, since TS(././AP) evaluates the whole neighborhood of an incumbent solution, whereas TS(././AP) only considers a subset of the neighborhood solutions.

7.2 Largest Gap-Routing

Solution Quality

For Largest Gap-Routing, TS(FCFS/$\mathcal{N}_{\text{com}}$/AP)(:=TS*(LGR)) provides the best solutions on average (average improvement: 4.1%, maximal improvement: 6.0% (40; 75), minimal improvement: 2.5% (40; 30)). In Table 10.3 in the appendix, the average improvements of the variants of TS are depicted for those problem classes where routing is done by application of the Largest Gap-Heuristic.
As for S-Shape-Routing, the neighborhood structure $\mathcal{N}_{\text{swap}}$ does not result to an improvement as long as the initial solution is generated by means of FCFS. In contrast to the results for S-Shape-Routing, independent of the initial solution, improvements of more than 2.0% can be observed for TS(./$\mathcal{N}_{\text{shift}}$/.). Differences obtained by application of the two initial solutions are only significant if the neighborhood structure $\mathcal{N}_{\text{swap}}$ is applied.
As for the three neighborhood structures, those variants of TS provide the largest improvements which include $\mathcal{N}_{\text{com}}$.
Concerning the size of the generated neighbourhood, the algorithms featuring AP and BI do not provide significantly different average improvements. It should be noted, that for problem classes with a small capacity of the picking device, AP results in more significant improvements than BI does (cf. problem class (100; 30)).

Computing Times

The average computing times for the best performing method TS*(LGR) range from 12 seconds (60; 30) up to 109 seconds (100; 75) per problem instance. The ratios of the computing times required by the different variants to each other is in the case of Largest Gap-Routing similar to the one observed for S-Shape-Routing (cf. Table 10.4 in the appendix).

8 Configuration Pretests for ABHC

8.1 S-Shape-Routing

Solution Quality

On average, of all variants of ABHC, ABHC(C&W(ii)/$\mathcal{N}_{\text{com}}$/$\mathbf{A}_{\text{o, o}}$)(:=ABHC*) provides the shortest total tour lengths (average improvement: 4.6%, maximal improvement 6.2%, minimal improvement: 3.7%). A detailed view of the average improvements over C&W(ii) for all considered configurations is given in the appendix (c.f. Table 10.5).
As has already been observed during the configuration pretests for TS, the impact of the initial solution is pivotal for the solution quality if $\mathcal{N}_{\text{shift}}$ or $\mathcal{N}_{\text{swap}}$ are used. If neighborhood solutions are obtained by shifts and swaps, the impact of the initial solution diminish.
For the three neighborhood structures, an increase in the capacity of the picking device reduces the improvements obtained by ABHC(./$\mathcal{N}_{\text{shift}}$/.), whereas the improvements obtained by ABHC(./$\mathcal{N}_{\text{swap}}$/.) tend to grow with an increase of the capacity. Nevertheless, ABHC(./$\mathcal{N}_{\text{shift}}$/.) and ABHC(./$\mathcal{N}_{\text{swap}}$/.) are outperformed by ABHC(./$\mathcal{N}_{\text{com}}$/.).
Application of ABHC(././$\mathbf{A}_{\text{o, o}}$) leads to higher improvements than ABHC(././$\mathbf{A}^{\text{lex}}_{\text{b, o}}$) does; the difference amounts approximately to an average of 0.4 percentage points.

Computing Times

The average computing times per problem instance for all considered variants of ABHC are presented in Table 10.6 in the appendix. Due to neglectable times for classes of small instances, only the results for problem classes with more than 40 customer orders are listed. For the best performing combination ABHC*, the computing times range from 13 seconds (60; 30) up to 188 seconds (100; 45).
Similar to the computing times of TS, those variants, which incorporate C&W(ii) for the generation of the initial solution, terminate faster than the corresponding variants which use FCFS.
If the neighborhood structure $\mathcal{N}_{\text{shift}}$ is used, ABHC(./$\mathcal{N}_{\text{shift}}$/.) stops after at most 14 seconds, whereas the other two neighborhood structures lead to computing times at least 10 times larger. Similar to the variants of TS, ABHC(./$\mathcal{N}_{\text{swap}}$/.) results in longer computing times than ABHC(./$\mathcal{N}_{\text{com}}$/.) does.
Comparing the two attribute sets, an increase of the capacity induces larger computing times of ABHC(././$\mathbf{A}_{\text{o, o}}$). Using the attribute set $\mathbf{A}^{\text{lex}}_{\text{b, o}}$, an increase of the capacity tends to reduce the computing times. This is due to the fact that a higher capacity reduces the number of necessary batches and, therefore, reduces the number of batch-order-combinations.

8.2 Largest Gap-Routing

Solution Quality

The improvements of the ABHC variants (cf. Table 10.7) for Largest Gap-Routing are of very similar size and structure as those for S-Shape-Routing. Therefore, a detailed description is omitted, here. Again, ABHC(C&W(ii)/$\mathcal{N}_{\text{com}}$/$\mathbf{A}_{\text{o, o}}$) can be identified as the best performing variant with an average improvement of 4.6% over to the solutions provided by C&W(ii). The improvements range from 3.5% (40; 30) to 6.1% (40; 75).

Computing Times

The computing times of the best performing ABHC variant vary between 28 (60; 30) and 500 (100; 75) seconds (cf. Table 10.8). As has already been observed for the solution quality, also for the computing times it can be stated that the ratios of the computing times required by the variants to each other remains valid, if the routes are determined by the Largest Gap-Heuristic.

9 Comparative Tests

9.1 Benchmarks

In order to evaluate the solution quality and the necessary computing times of the best performing Tabu Search and Attributed-Based Hill Climber variants, the performance of the proposed methods is compared to that of several other batching strategies.
In order to provide a classical local search method against which TS and ABHC can be benchmarked, we combine C&W(ii) with a simple local search procedure. In this local search procedure the neighborhood structure $\mathcal{N}_{\text{com}}$ is used and a best improvement strategy is applied. This method will be denoted as LS, i.e. LS:=LS(C&W(ii)/$\mathcal{N}_{\text{com}}$/BI)).
As a further benchmark we consider the Iterated Local Search approach of Henn et al. (2009). The heuristic consists of two alternating phases, a local search and a perturbation phase. A first incumbent solution is generated by means of FCFS and then improved by a local search phase. This phase is characterized by the application of a variable neighborhood descent method with two neighborhood structures ($\mathcal{N}_{\text{shift}}$ and $\mathcal{N}_{\text{swap}}$) and a first improvement strategy (FI). The solution stemming from this local search phase has to pass an acceptance criterion in order to become the next incumbent solution, otherwise the previous solution remains the incumbent solution. This solution is partially modified in the perturbation phase by randomly chosen exchanges of customer orders assigned to different batches. This solution serves as initial solution for the next local search phase. For a fair comparison we consider a different termination condition as proposed by Henn et al. (2009). After n local search phases without an improvements of the best known total tour lengths, a deterioration of the objective function value of the incumbent solution can be accepted. The algorithm terminates after 10 deteriorations. All other parameters of the algorithm are chosen identically to Henn et al. (2009). This algorithm is denoted by ILS-1,

Routing-Scheme	n	C	∅ no. of batches	no. of opt. sol.	∅ Gap [%]
S-Shape	40	30	2436	40	0.00
		45	39295	39	1.25
		60	354256	16	3.19
		75	-	-	-
Largest Gap	40	30	2731	40	0.00
		45	32147	37	1.13
		60	465547	15	2.26
		75	-	-	-

Table 9.1: Optimal Solutions for S-Shape- and Largest-Gap-Routing

i.e. ILS-1:=ILS(FCFS/$\mathcal{N}_{\text{com}}$/FI).
Furthermore, in order to provide another benchmark a second variant of ILS (denoted by ILS-2) has been implemented. Whereas the perturbation phase, the acceptance criterion and the termination condition of ILS-1 remain unchanged, the initial solution is generated by means of C&W(ii). In the local search phase the neighborhood structure $\mathcal{N}_{\text{com}}$ is used and a best improvement strategy is applied, i.e. ILS-2:=ILS(C&W(ii)/$\mathcal{N}_{\text{com}}$/BI).
In order to provide further insights into the solution quality of TS and ABHC, we have also tried to benchmark their solutions against the corresponding optimal objective function values. Whenever possible, the respective Integer Program (cf. Section 2) was generated and solved by CPLEX 10.1. This approach was only successful for a limited number of instances of the problem classes with $n = 40$. Table 9.1 depicts the results obtained for these problem classes. It is shown for how many instances CPLEX was able to generate a solution and prove its optimality (no. of opt. sol.). For the remaining instances CPLEX was not able to prove the optimality of the solution due to memory restrictions. For these instances the average gap between the lower bound and the best found objective function value is presented. Furthermore, the average number of feasible batches per problem instance is given, which corresponds to the number of columns of the mathematical model.

9.2 S-Shape-Routing

Solution Quality

In Table 9.2 the solution quality of the best performing TS and ABHC configurations (TS*(SSR) = TS(C&W(ii)/$\mathcal{N}_{\text{com}}$/AP); ABHC*=ABHC(C&W(ii)/$\mathcal{N}_{\text{com}}$/$\mathbf{A}_{\text{o, o}}$)) is compared to the results of the benchmark heuristics. For the problem classes in which the routing schemes are provided by the S-Shape-Heuristic, LS – in comparison to C&W(ii) – achieves a reduction of the total tour length of less than one percentage point on average. The reduction varies from 0.2% (80; 30) to 1.9% (40; 60). Therefore, it can be concluded that the solutions provided by C&W(ii) are already close to local minima and a significant improvement by means of a simple Local Search algorithm cannot be obtained.
By application of the two proposed implementations of ILS, total tour lengths are obtained which, on average, are 3.0% (ILS-1) and 3.2% (ILS-2) shorther than the tour lengts of the solutions

provided by C&W(ii). For ILS-1 these improvements vary between 1.2% (100; 60) and 5.1% (40; 75). The smallest reduction obtained by ILS-2 amounts to 1.6% (100; 60), and the largest one to 5.2% (40; 75). With respect to these small differences, it can be concluded that the impact of the exploration of the neighborhood, which is different in both applications, is neglectable. With respect to the proposed benchmark heuristics, significantly smaller tour lengths can be obtained by application of TS⋆(SSR) and ABHC⋆. In comparison to ILS, on average, additional improvements of 0.9 percentage points can be achieved by TS⋆(SSR) and 1.4 percentage points by ABHC⋆. For TS⋆(SSR) these improvements are larger than -0.7 percentage points (60; 30) and smaller than 1.7 percentage points (100; 60). The additional improvements of ABHC⋆ exceed 0.6 percentage points (40; 75) and are smaller than 2.1 percentage points (100; 60).

Comparing TS⋆(SSR) and ABHC⋆ to each other exclusively ABHC⋆ provides smaller total tour lengths on average. For 15 problem classes, ABHC⋆ finds better solutions than TS⋆(SSR) does. The largest deviations between both approaches occur for small capacities of the picking device (cf. problem classes with $C = 30$).

Table 9.3 relates the solution quality of TS⋆(SSR) and ABHC⋆ to the corresponding optimal objective function values. For all problem classes, TS⋆(SSR) and ABHC⋆ provide solutions close to the optimum. The average deviation from the optimal objective function value (related to the instances in each class which could be solved to a proven optimum) is smaller than two percent. For ABHC⋆ the average deviation from the optimal objective function value tends to be smaller for problems with a smaller capacity of the picking device than for those with a larger capacity. In the brackets of the third and forth column of Table 9.3 the average deviations between the objective function values provided by TS⋆/ABHC⋆and those obtained by CPLEX are presented. These entries includue those instances in which the optimality of the obtained solution could not

n	C	LS	ILS-1	ILS-2	TS⋆(SSR)	ABHC⋆
40	30	0.3	3.1	2.8	2.3	**3.8**
	45	1.1	4.5	4.3	**5.3**	**5.3**
	60	1.9	5.1	5.0	**6.2**	**6.2**
	75	1.2	5.0	5.2	5.6	**5.8**
60	30	0.3	3.4	3.5	2.8	**4.3**
	45	0.6	2.7	3.0	4.0	**4.4**
	60	0.8	2.9	3.4	4.6	**4.8**
	75	1.2	3.5	4.0	5.2	**5.4**
80	30	0.2	2.9	3.0	2.4	**3.7**
	45	0.6	2.3	2.4	3.9	**4.2**
	60	0.7	2.1	2.4	4.0	**4.2**
	75	1.1	2.5	3.0	**4.5**	**4.5**
100	30	0.2	3.0	3.4	3.1	**4.2**
	45	0.4	2.2	2.3	4.0	**4.4**
	60	0.6	1.2	1.6	3.3	**3.7**
	75	0.7	1.2	2.4	4.1	**4.2**
Average		0.8	3.0	3.2	4.1	**4.6**
Maximum:		1.9	5.1	5.2	**6.2**	**6.2**
Minimum		0.2	1.2	1.6	2.3	**3.7**

Table 9.2: Improvement of the average tour length by TS⋆(SSR), ABHC⋆, and benchmark heuristics [in %] in comparison to C&W(ii) for S-Shape-Routing

n	C	dev. of TS*(SSR)	dev. of ABHC*
40	30	1.61 (1.61)	0.09 (0.09)
	45	0.78 (0.76)	0.74 (0.73)
	60	1.74 (0.74)	1.43 (0.72)
	75	-	-

Table 9.3: Average deviation of the total tour length obtained by TS*(SSR)/ABHC* from the optimal total tour length [in %]; in brackets: average deviation of the objective function value of TS*(SSR)/ABHC* from the objective function value provided by CPLEX for S-Shape-Routing

be proven. For some of these instances, the heuristics generate solutions with a smaller total tour lengths than those provided by CPLEX. Therefore, the obtained average deviation is smaller than the deviation between the total tour lengths provided by TS*(SSR)/ABHC* and the optimal ones.

Computing Times

The average computing times per problem instance for the benchmark heuristics, TS*(SSR) and ABHC* are presented in Table 9.4. For ILS-1 the time varies between 9 seconds (60; 30) and 155 seconds (100; 75), and for ILS-2 between 19 seconds (60; 30) and 246 seconds (100; 45). Due to the exploration of the entire neighborhood, ILS-2 proves to be the most time consuming variant of ILS.
Comparing the computing times of ILS and TS*(SSR) to each other consumes up to 50% of the computing times required by ILS and provides better solutions than ILS does. The differences between ILS and ABHC* are not significant (up to 60 seconds) and involve higher quality solutions of ABHC*. TS*(SSR) requires significantly smaller computing times than ABHC* (around 20% for the problem classes with 100 customer orders). Comparing the computing times of TS*(SSR) and ABHC*, the TS variant already terminates after 34% of the computing time of the best ABHC variant, on average
It can be concluded that for a large capacity of the picking device, TS*(SSR) provides similar,

n	C	LS	ILS-1	ILS-2	TS*(SSR)	ABHC*
60	30	0	9	19	6	13
	45	0	18	33	11	22
	60	0	21	30	14	22
	75	0	22	34	14	25
80	30	0	31	46	11	45
	45	0	62	91	24	78
	60	0	64	89	22	69
	75	0	68	96	24	82
100	30	0	68	135	21	126
	45	1	129	246	38	188
	60	1	138	170	35	178
	75	1	155	173	42	180

Table 9.4: Average computing times [in sec.] per problem instance of TS*(SSR), ABHC*, and benchmark heuristics for S-Shape-Routing

high quality solutions in shorter computing times than ABHC⋆does, whereas, for small capacities, the larger computing times of ABHC⋆ are justified by a significantly larger improvement of the total tour length.

9.3 Largest Gap-Routing

Solution Quality

n	C	LS	ILS-1	ILS-2	TS⋆(LGR)	ABHC⋆
40	30	0.3	2.9	2.4	1.1	**3.5**
	45	0.6	3.1	3.1	**3.8**	**3.8**
	60	1.0	4.4	4.2	**4.8**	**4.8**
	75	1.8	5.7	5.5	**6.2**	6.1
60	30	0.2	2.8	2.6	1.1	**3.7**
	45	0.5	2.9	2.9	3.6	**3.7**
	60	0.9	4.1	4.4	5.0	**5.1**
	75	1.4	5.3	5.5	**6.2**	6.1
80	30	0.1	3.0	2.9	0.3	**4.0**
	45	0.5	2.5	2.8	3.2	**3.7**
	60	0.7	3.4	3.6	4.4	**4.5**
	75	1.0	4.5	4.9	5.5	**5.8**
100	30	0.1	2.9	3.0	0.8	**4.2**
	45	0.4	2.6	2.7	3.7	**3.9**
	60	0.6	3.0	3.2	**4.4**	**4.4**
	75	0.9	4.6	4.9	5.5	**5.8**
Average		0.7	3.6	3.7	3.7	**4.6**
Maximum:		1.8	5.7	5.5	**6.2**	6.1
Minimum		0.1	2.5	2.4	0.3	**3.5**

Table 9.5: Improvement of the average tour length by TS⋆(LGR), ABHC⋆, and benchmark heuristics [in %] in comparison to C&W(ii) for Largest Gap-Routing

Table 9.5 compares the best performing TS and ABHC variants (TS⋆(LGR)=TS(FCFS/$\mathcal{N}_{\text{com}}$/BI); ABHC⋆ = ABHC(C&W(ii)/$\mathcal{N}_{\text{com}}$/$\mathbf{A}_{\text{o, o}}$)) to the benchmark heuristics for Largest Gap-Routing. The improvements of C&W(ii)+LS as compared to C&W(ii) are of similar size as in the case of S-Shape-Routing. Likewise the ratios of the improvements among the ILS variants but also between the ILS variants and ABHC⋆ are almost identical to the S-Shape case. Consequently, details will not be presented here. On average, the application of TS⋆(LGR) leads to an improvement similar to that of ILS (TS⋆(LGR): 3.7%; ILS-1: 3.6%; ILS-2: 3.7%), even though the spread of the improvements may differ significantly across the problem classes. For a small capacity ($C = 30$), TS⋆(LGR) is not able to provide improvements of more than 1.1%, whereas, for a large capacity ($C = 100$), the improvements are the best ones observed at all. ABHC⋆ outperforms TS⋆(LGR) in 12 problem classes.

The solution quality of TS⋆(LGR) and ABHC⋆ is presented in Table 9.6, which depicts the average deviation of the obtained total tour lengths from the optimal ones. Similar to the case of S-Shape-Routing, the average deviation is small for ABHC⋆ (less than 0.4%). For a large capacity, TS⋆(LGR) provides solutions with tour lengths, which differ at most 0.73% from the

n	C	TS*(LGR)	ABHC*
40	30	2.74 (2.74)	0.24 (0.24)
	45	0.37 (0.34)	0.39 (0.36)
	60	0.73 (0.14)	0.38 (0.09)
	75	-	-

Table 9.6: Average deviation of the total tour length obtained by TS*(SSR)/ABHC* from the optimal total tour length [in %]; in brackets: average deviation of the objective function value of TS*(LGR)/ABHC* from the objective function value provided by CPLEX for Largest Gap-Routing

optimal objective function values. In class (40; 30), the difference of TS*(LGR) to the optimal total tour lengths amounts to 2.74%.

Computing Times

The average computing times are shown in Table 9.7, where only the results for problem classes with more than 40 customer orders are presented. Due to the higher computational effort required by the Largest Gap-Heuristic, these computing times are larger than in the case of S-Shape-Routing. Among the investigated metaheuristics TS*(LGR) provides solutions in the shortest time, whereas ABHC* requires computing times of up to 500 seconds.

n	C	LS	ILS-1	ILS-2	TS*(LGR)	ABHC*
60	30	0	21	32	12	28
	45	0	43	65	20	52
	60	0	52	71	25	55
	75	1	60	80	30	57
80	30	0	71	118	27	106
	45	1	118	209	41	167
	60	1	151	232	47	174
	75	1	169	273	62	178
100	30	1	166	281	47	317
	45	1	299	511	61	424
	60	2	322	491	76	432
	75	2	397	569	109	500

Table 9.7: Average computing times [in sec.] per problem instance of TS*(LGR), ABHC*, and benchmark heuristics for Largest Gap-Routing

10 Conclusions and Outlook

This paper dealt with the Order Batching Problem, a problem pivotal for the efficient management and control of manual picker-to-parts order picking systems in distribution warehouses. For the solution of this problem two metaheuristics have been applied, namely Tabu Search and the Attribute-Based Hill Climber. For each of these two heuristics we analyzed the performance of different configurations in extensive numerical experiments. By means of comparing the proposed methods to selected benchmark heuristics it was shown that both approaches generate improved solutions of similar quality. Furthermore, the computing times of both approaches are significantly smaller than the ones of existing solution approaches, which provide similar total tour lengths. Therefore, both variants appear very suitable for being implemented in software systems for practical purposes. The application of both approaches can reduce the total tour length, and consequently result in a reduction of the overall picking time. This might allow even for a reduction of overtimes or for a reduction of the workforce, both critical aspects in warehouse environments where low profit margins prevail. Furthermore, the improved solutions can result in shorter lead times in picking customer orders, and, thus, in an improved customer service.

Future research should concentrate on the adaption of these powerful metaheuristics for dynamic warehouse environments. In theses situations, customer orders arrive over time and a huge number of batching steps is necessary.

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Appendix

		FCFS						**C&W (ii)**					
		$\mathcal{N}_{\text{shift}}$		$\mathcal{N}_{\text{swap}}$		$\mathcal{N}_{\text{com}}$		$\mathcal{N}_{\text{shift}}$		$\mathcal{N}_{\text{swap}}$		$\mathcal{N}_{\text{com}}$	
n	C	BI	AP	BI	AP	BI	AP	BI	AP	BI	AP	BI	AP
40	30	0.5	1.2	-10.6	-9.9	1.5	**2.8**	1.6	1.6	1.3	1.4	2.0	2.3
	45	-0.7	-0.4	-3.7	-3.8	5.1	5.0	3.0	3.0	2.9	2.9	5.2	**5.3**
	60	1.0	0.9	-0.2	-0.1	5.9	6.2	3.4	3.5	4.0	4.1	**6.3**	6.2
	75	1.4	1.4	0.4	0.8	5.2	**5.6**	3.3	3.2	4.5	4.6	**5.6**	**5.6**
60	30	1.4	1.3	-11.8	-10.8	2.2	3.2	1.8	1.8	1.2	1.4	2.3	**2.8**
	45	-1.9	-1.1	-3.8	-3.6	3.7	**4.0**	1.9	2.0	2.5	2.7	**4.0**	**4.0**
	60	-1.0	-0.7	-2.5	-2.2	4.1	4.6	2.5	2.6	3.4	3.4	**4.7**	4.6
	75	-0.3	-0.5	-0.7	-0.4	4.4	4.9	2.7	2.7	4.1	3.9	**5.3**	5.2
80	30	0.8	0.5	-12.7	-11.6	1.2	**2.6**	1.5	1.6	0.9	1.3	1.5	2.4
	45	-2.8	-2.0	-5.1	-4.9	3.6	3.8	1.9	1.9	2.3	2.4	3.8	**3.9**
	60	-2.1	-1.7	-3.2	-3.0	3.1	3.7	2.1	1.8	2.8	2.6	3.9	**4.0**
	75	-0.7	-0.2	-2.0	-1.6	3.6	3.9	2.8	2.7	3.7	3.5	4.4	**4.5**
100	30	0.9	1.1	-12.0	-10.8	1.6	**3.4**	1.7	2.0	1.1	1.6	2.1	3.1
	45	-3.9	-2.7	-4.7	-4.5	3.7	3.8	2.1	2.1	2.2	2.4	3.8	**4.0**
	60	-3.2	-2.4	-3.3	-3.2	2.1	3.2	1.8	2.0	2.4	2.4	**3.4**	3.3
	75	-3.6	-2.7	-2.1	-1.7	2.6	3.5	2.4	2.2	3.1	3.0	3.8	**4.1**
Average		-0.9	-0.5	-4.9	-4.4	3.3	4.0	2.3	2.3	2.6	2.7	3.9	**4.1**
Maximum:		1.4	1.4	0.4	0.8	5.9	**6.2**	3.4	3.5	4.5	4.6	4.6	**6.2**
Minimum		-3.9	-2.7	-12.7	-11.6	1.2	**2.6**	1.5	1.6	0.9	1.3	1.3	2.3

Table 10.1: Improvement of the average tour length by TS [in %] in comparison to C&W(ii) for S-Shape-Routing

		FCFS						**C&W (ii)**					
		$\mathcal{N}_{\text{shift}}$		$\mathcal{N}_{\text{swap}}$		$\mathcal{N}_{\text{com}}$		$\mathcal{N}_{\text{shift}}$		$\mathcal{N}_{\text{swap}}$		$\mathcal{N}_{\text{com}}$	
n	C	BI	AP	BI	AP	BI	AP	BI	AP	BI	AP	BI	AP
60	30	10	4	16	5	23	7	3	3	12	7	14	6
	45	10	7	37	11	30	11	4	4	20	12	24	11
	60	11	10	48	13	41	14	5	6	29	15	28	14
	75	12	11	62	16	55	15	6	6	35	14	35	14
80	30	25	7	37	9	58	12	7	7	28	13	33	11
	45	23	15	107	21	82	23	7	7	56	25	65	24
	60	26	19	122	26	90	28	10	9	67	26	61	22
	75	30	27	168	30	159	29	12	12	93	28	89	24
100	30	48	11	74	17	127	23	14	11	60	26	71	21
	45	39	27	244	34	183	38	13	13	123	42	111	38
	60	51	35	279	42	199	43	16	15	147	39	144	35
	75	48	39	344	52	279	46	20	21	161	48	159	42

Table 10.2: Average computing times [in sec.] per problem instance for TS and S-Shape-Routing

		FCFS						C&W (ii)					
		$\mathcal{N}_{\text{shift}}$		$\mathcal{N}_{\text{swap}}$		$\mathcal{N}_{\text{com}}$		$\mathcal{N}_{\text{shift}}$		$\mathcal{N}_{\text{swap}}$		$\mathcal{N}_{\text{com}}$	
n	C	BI	AP	BI	AP	BI	AP	BI	AP	BI	AP	BI	AP
40	30	0.7	2.3	-9.1	-8.7	1.1	**2.5**	1.2	1.0	0.7	1.0	1.2	1.6
	45	2.1	2.0	-3.6	-3.6	**3.8**	**3.8**	2.6	2.7	1.9	1.8	**3.8**	**3.8**
	60	2.5	2.6	-0.4	-0.5	**4.8**	4.6	3.3	3.2	3.5	3.3	**4.8**	4.7
	75	4.5	4.5	2.2	2.1	**6.2**	6.0	4.8	4.7	5.0	5.0	**6.2**	6.1
60	30	0.7	2.4	-9.5	-9.2	1.1	**2.9**	1.1	0.9	0.5	0.8	0.6	1.3
	45	0.9	1.2	-3.6	-3.7	**3.6**	3.5	2.0	2.0	2.3	2.3	**3.6**	3.5
	60	2.4	2.3	0.1	-0.2	**5.0**	4.8	3.2	3.2	3.6	3.5	**5.0**	4.8
	75	3.9	3.8	1.6	1.4	6.2	5.8	4.0	4.2	4.7	4.5	**6.3**	5.8
80	30	-0.2	2.8	-11.3	-10.9	0.3	**2.9**	0.8	0.7	0.5	0.9	0.8	1.4
	45	0.0	0.5	-4.1	-4.3	3.2	3.3	2.0	2.0	2.0	1.9	**3.4**	**3.4**
	60	1.3	1.2	-1.0	-1.5	4.4	4.2	2.6	2.5	3.3	3.2	**4.5**	4.3
	75	2.3	2.3	1.3	0.8	5.5	5.3	3.6	3.6	4.8	4.6	**5.7**	5.4
100	30	0.4	2.9	-10.7	-10.3	0.8	**3.3**	0.8	1.0	0.5	1.1	0.8	1.7
	45	0.6	0.6	-3.9	-4.2	3.7	3.4	1.9	1.8	2.2	2.1	**3.8**	3.6
	60	0.4	0.7	-1.0	-1.5	4.4	3.9	2.5	2.5	3.1	2.9	**4.4**	4.1
	75	2.0	2.0	1.4	0.9	5.5	5.3	3.6	3.7	4.8	4.6	**5.8**	5.4
Average		1.5	2.1	-3.2	-3.3	3.7	**4.1**	2.5	2.5	2.7	2.7	3.8	3.8
Maximum:		4.5	4.5	2.2	2.1	6.2	6.0	4.8	4.7	5.0	5.0	**6.3**	6.1
Minimum		-0.2	0.5	-11.3	-10.9	0.3	**2.5**	0.8	0.7	0.5	0.8	0.6	1.3

Table 10.3: Improvement of the average tour length by TS [in %] in comparison to C&W(ii) Largest Gap-Routing

		FCFS						C&W (ii)					
		$\mathcal{N}_{\text{shift}}$		$\mathcal{N}_{\text{swap}}$		$\mathcal{N}_{\text{com}}$		$\mathcal{N}_{\text{shift}}$		$\mathcal{N}_{\text{swap}}$		$\mathcal{N}_{\text{com}}$	
n	C	BI	AP	BI	AP	BI	AP	BI	AP	BI	AP	BI	AP
60	30	21	11	28	8	40	12	8	7	24	12	26	10
	45	27	21	80	19	66	20	11	8	47	22	49	17
	60	26	22	106	27	80	25	14	12	66	30	62	24
	75	26	23	131	30	96	30	16	14	77	33	68	27
80	30	49	24	65	17	91	27	16	12	51	26	57	21
	45	66	40	227	34	154	41	17	18	131	42	108	41
	60	63	46	285	45	193	47	24	21	145	54	147	47
	75	66	54	307	55	236	62	34	28	189	65	167	58
100	30	88	37	127	26	184	47	28	23	104	47	117	34
	45	150	59	446	53	336	61	33	29	232	69	234	64
	60	98	86	519	69	382	76	37	39	309	82	254	78
	75	120	103	610	97	486	109	54	50	372	103	344	88

Table 10.4: Average computing times [in sec.] per problem instance for TS and Largest Gap-Routing

		FCFS						**C&W (ii)**					
		$\mathcal{N}_{\text{shift}}$		$\mathcal{N}_{\text{swap}}$		$\mathcal{N}_{\text{com}}$		$\mathcal{N}_{\text{shift}}$		$\mathcal{N}_{\text{swap}}$		$\mathcal{N}_{\text{com}}$	
n	C	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}_{\text{b,o}}^{\text{lex}}$	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}_{\text{b,o}}^{\text{lex}}$	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}_{\text{b,o}}^{\text{lex}}$	$\mathbf{A}_{\text{o, o}}$	$\mathbf{A}_{\text{b,o}}^{\text{lex}}$	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}_{\text{b,o}}^{\text{lex}}$	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}_{\text{b,o}}^{\text{lex}}$
40	30	1.7	1.8	-9.2	-9.2	**3.8**	3.7	2.1	2.4	1.9	1.8	**3.8**	**3.8**
	45	-1.8	-2.0	-3.5	-3.7	**5.5**	5.0	2.1	2.4	3.1	2.8	5.3	5.1
	60	-0.5	-1.2	-0.1	-0.5	**6.3**	5.6	2.7	2.6	4.1	3.8	6.2	5.6
	75	0.4	-0.8	0.9	-0.1	5.5	4.7	2.7	2.4	5.0	4.2	**5.8**	4.9
60	30	1.6	1.9	-10.2	-10.2	**4.3**	4.2	2.7	2.6	2.2	2.1	**4.3**	4.2
	45	-4.0	-3.8	-3.2	-3.6	4.6	4.0	1.6	1.7	3.0	2.7	**4.4**	3.9
	60	-2.1	-2.5	-1.7	-2.3	4.9	4.2	1.9	1.9	3.6	3.3	**4.8**	4.5
	75	-2.4	-2.2	0.1	-0.6	5.2	4.0	2.3	2.2	4.2	3.7	**5.4**	4.9
80	30	1.2	1.7	-11.0	-11.0	**3.7**	**3.7**	2.0	2.3	2.0	1.9	**3.7**	3.6
	45	-4.7	-4.1	-4.4	-4.7	**4.2**	4.0	1.4	1.4	2.8	2.5	**4.2**	4.0
	60	-3.8	-3.2	-2.0	-2.7	4.1	3.4	1.5	1.5	3.1	2.8	**4.2**	4.0
	75	-3.2	-3.4	-0.6	-1.6	**4.5**	3.4	2.3	2.1	4.0	3.5	**4.5**	4.2
100	30	1.7	2.0	-10.2	-10.3	**4.2**	**4.2**	2.2	2.5	2.2	2.1	**4.2**	**4.2**
	45	-5.2	-4.9	-4.0	-4.2	4.3	4.0	1.5	1.7	2.7	2.4	**4.4**	4.0
	60	-5.4	-4.8	-2.2	-2.9	3.5	2.9	1.3	1.6	2.9	2.5	**3.7**	3.4
	75	-4.4	-4.7	-0.6	-1.5	3.8	2.8	1.7	1.7	3.4	3.0	**4.2**	3.9
Average		-1.9	-1.9	-3.9	-4.3	4.5	4.0	2.0	2.1	3.1	2.8	**4.6**	4.3
Maximum:		1.7	2.0	0.9	-0.1	**6.3**	5.6	2.7	2.6	5.0	4.2	6.2	5.6
Minimum		-5.4	-4.9	-11.0	-11.0	3.5	2.8	1.3	1.4	1.9	1.8	**3.7**	3.4

Table 10.5: Improvement of the average tour length by ABHC [in %] in comparison to C&W(ii) for S-Shape-Routing

		FCFS						**C&W (ii)**					
		$\mathcal{N}_{\text{shift}}$		$\mathcal{N}_{\text{swap}}$		$\mathcal{N}_{\text{com}}$		$\mathcal{N}_{\text{shift}}$		$\mathcal{N}_{\text{swap}}$		$\mathcal{N}_{\text{com}}$	
n	C	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}_{\text{b,o}}^{\text{lex}}$	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}_{\text{b,o}}^{\text{lex}}$	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}_{\text{b,o}}^{\text{lex}}$	$\mathbf{A}_{\text{o, o}}$	$\mathbf{A}_{\text{b,o}}^{\text{lex}}$	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}_{\text{b,o}}^{\text{lex}}$	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}_{\text{b,o}}^{\text{lex}}$
60	30	2	1	22	36	22	14	1	2	14	18	13	15
	45	1	1	47	25	30	12	0	1	26	13	22	11
	60	2	1	52	25	33	13	1	1	27	11	22	9
	75	2	1	55	19	35	13	1	1	29	10	25	10
80	30	7	7	81	154	78	61	2	7	44	64	45	58
	45	2	1	172	104	106	46	1	1	101	49	78	36
	60	5	3	204	97	103	50	2	2	96	35	69	29
	75	7	3	226	83	124	57	4	2	107	30	82	27
100	30	13	12	213	421	199	159	5	14	124	179	126	167
	45	3	2	432	289	274	108	1	2	257	137	188	100
	60	5	4	543	230	276	118	3	3	237	86	178	75
	75	14	6	608	219	316	122	8	5	244	72	180	61

Table 10.6: Average computing times [in sec.] per problem instance for ABHC and S-Shape-Routing

		FCFS						C&W (ii)					
		$\mathcal{N}_{\text{shift}}$		$\mathcal{N}_{\text{swap}}$		$\mathcal{N}_{\text{com}}$		$\mathcal{N}_{\text{shift}}$		$\mathcal{N}_{\text{swap}}$		$\mathcal{N}_{\text{com}}$	
n	C	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}^{\text{lex}}_{\text{b,o}}$	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}^{\text{lex}}_{\text{b,o}}$	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}^{\text{lex}}_{\text{b,o}}$	$\mathbf{A}_{\text{o, o}}$	$\mathbf{A}^{\text{lex}}_{\text{b,o}}$	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}^{\text{lex}}_{\text{b,o}}$	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}^{\text{lex}}_{\text{b,o}}$
40	30	2.3	2.4	-8.2	-8.2	**3.5**	**3.5**	2.5	2.7	1.4	1.4	**3.5**	3.4
	45	-0.6	-0.8	-3.5	-3.7	**3.8**	3.6	1.8	1.7	2.0	1.8	**3.8**	3.7
	60	-0.7	-1.2	-0.4	-0.7	**4.8**	4.4	2.0	1.9	3.5	3.2	**4.8**	4.6
	75	2.8	1.8	2.3	2.0	6.0	5.5	3.7	3.5	4.9	4.6	**6.1**	5.7
60	30	2.3	2.6	-8.6	-8.6	**3.7**	**3.7**	2.6	2.9	1.3	1.3	**3.7**	3.6
	45	-1.7	-1.5	-3.4	-3.6	**3.8**	3.5	1.4	1.3	2.4	2.3	3.7	3.5
	60	0.1	0.0	0.1	-0.2	5.0	4.6	2.4	2.4	3.6	3.3	**5.1**	4.8
	75	1.3	1.2	1.8	1.2	**6.1**	5.8	3.5	3.3	4.7	4.4	**6.1**	5.8
80	30	2.2	2.7	-10.2	-10.2	**4.0**	**4.0**	2.7	2.9	1.4	1.4	**4.0**	3.9
	45	-2.6	-2.8	-3.9	-4.0	**3.7**	3.5	1.4	1.5	2.2	2.0	**3.7**	3.5
	60	-1.1	-1.6	-0.9	-1.1	**4.5**	4.2	2.0	1.7	3.4	3.1	**4.5**	4.2
	75	0.7	0.1	1.5	1.0	5.6	5.2	2.9	2.6	4.9	4.6	**5.8**	5.5
100	30	2.5	2.8	-9.6	-9.6	**4.2**	4.1	2.7	3.1	1.5	1.5	**4.2**	**4.2**
	45	-1.8	-2.0	-3.7	-3.9	**3.9**	3.7	1.3	1.4	2.4	2.2	**3.9**	3.8
	60	-1.9	-2.0	-0.7	-1.0	**4.5**	4.3	1.9	1.9	3.3	3.0	4.4	4.3
	75	-0.1	-0.5	1.7	1.2	**5.8**	5.2	2.7	2.5	5.0	4.6	**5.8**	5.5
Average		0.2	0.1	-2.9	-3.1	**4.6**	4.3	2.3	2.3	3.0	2.8	**4.6**	4.4
Maximum:		2.8	2.8	2.3	2.0	**6.1**	5.8	3.7	3.5	5.0	4.6	**6.1**	5.8
Minimum		-2.6	-2.8	-10.2	-10.2	**3.5**	**3.5**	1.3	1.3	1.3	1.3	**3.5**	3.4

Table 10.7: Improvement of the average tour length by ABHC [in %] in comparison to C&W(ii) for Largest Gap-Routing

		FCFS						C&W (ii)					
		$\mathcal{N}_{\text{shift}}$		$\mathcal{N}_{\text{swap}}$		$\mathcal{N}_{\text{com}}$		$\mathcal{N}_{\text{shift}}$		$\mathcal{N}_{\text{swap}}$		$\mathcal{N}_{\text{com}}$	
n	C	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}^{\text{lex}}_{\text{b,o}}$	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}^{\text{lex}}_{\text{b,o}}$	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}^{\text{lex}}_{\text{b,o}}$	$\mathbf{A}_{\text{o, o}}$	$\mathbf{A}^{\text{lex}}_{\text{b,o}}$	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}^{\text{lex}}_{\text{b,o}}$	$\mathbf{A}_{\text{o,o}}$	$\mathbf{A}^{\text{lex}}_{\text{b,o}}$
60	30	5	5	45	71	40	35	2	5	27	37	28	34
	45	2	1	103	60	65	29	1	1	56	29	52	26
	60	5	2	102	45	63	25	3	2	63	24	55	24
	75	6	3	106	41	64	24	4	3	67	25	57	22
80	30	12	9	158	281	160	125	5	9	98	121	106	114
	45	5	2	351	184	219	82	2	3	213	98	167	75
	60	9	4	382	168	204	76	5	4	216	83	174	75
	75	20	9	386	163	217	80	12	7	223	80	178	71
100	30	31	24	413	766	448	354	16	37	270	348	317	355
	45	12	6	880	498	569	214	4	6	551	264	424	205
	60	17	10	991	458	543	213	8	8	537	211	432	174
	75	42	16	1.059	432	603	206	18	10	601	211	500	178

Table 10.8: Average computing times [in sec.] per problem instance for ABHC and Largest Gap-Routing