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A U-Shaped Layout for a Manual Order Picking System

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Abstract

In manual order picking systems, order pickers walk or ride through a warehouse in order to collect items requested by customers. The performance of such a system is significantly dependent on its layout which determines the lengths of the order pickers' tours and the corresponding picking times. Whereas for classic warehouse layouts all picking aisles are arranged in parallel to each other, in the warehouse layout presented here the picking aisles are arranged around a U-shaped central aisle. This layout has been developed for order picking systems in which slow-moving items are prevalent. A new routing strategy for such a warehouse is presented and an analytical expression for the expected tour length per picking order is derived. By comparing this estimation with those of routing schemes in classic warehouse layouts, it is demonstrated in which situations such U-shaped layouts allow for operating warehouses more efficiently.

Keywords: Logistics, Warehouse Management, Order Picking, Layout, Picker Routing

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1 Introduction

Order picking is a warehouse function dealing with the retrieval of items from their storage locations in order to satisfy a given demand specified by customer requests (Petersen and Schmenner, 1999). Order picking arises because incoming items are received and stored in (large-volume) unit loads while (internal or external) customers order small volumes (less-than-unit loads) of different item types. Order picking is critical to each supply chain, since underperformance results in an unsatisfactory customer service (long processing and delivery times, incorrect shipments) and high costs (labour cost, cost of additional and/or emergency shipments).

Even though there have been attempts to automate the picking process, systems involving human operators are still prevalent in practice. Such manual order picking systems can be differentiated into two categories (Wäscher, 2004): Picker-to-parts systems, where order pickers walk or ride through the warehouse and collect the required items; and parts-to-picker systems, where automated storage and retrieval systems deliver the items to stationary pickers.

In the past, research on order picking systems has focused on distribution warehouses, in which fast moving items are dealt with. Order picking for slow-moving items, characterized by low (probably close to zero) annual demands, has been neglected in the literature so far. Corresponding warehouses (e.g. for spare parts) are different to distribution warehouses. In particular, they are designed in order to make efficiently use of the available space in the first place. Consequently, the layouts of such warehouses are unique and the routing of order pickers is limited to very specific schemes. Research findings from traditional distribution warehouses, therefore, cannot immediately be transferred to warehouses for slow-moving items.

In this paper, we will be studying three different layout types of warehouses for slow-moving items and analyze their efficiency. The remainder of this paper is organized as follows: In Section 2 the order picking process and general requirements concerning the layout of order-picking warehouses for slow-moving items will be described. Furthermore, a layout type which has been proposed recently, the so-called U-shaped layout and the corresponding routing strategy will be presented in this section. Also two reference layouts will be introduced. Section 3 contains an overview of the relevant literature. Expressions for the expected lengths of picking tours in the layouts under discussion will be derived in Section 4. These tour lengths will be verified by means of numerical experiments in Section 5. In Section 6 the derived analytic expressions will be used in order to identify under which conditions one layout type outperforms the others.

2 Order Picking in Warehouses for Slow-Moving Items

2.1 The Picking Process

Typically, the picking area of an order-picking warehouse consists of a number of aisles where item types (articles) are stored – either on racks, pallets or directly on the floor – on both sides of the aisles (Ballou, 1967). Order pickers walk (or ride) through the warehouse in order to collect items requested by internal or external customers. They start at the depot, travel through the picking area, stop at the storage locations of the respective articles, remove the required article quantities, and return to the depot where they hand in the picked items.

In order to avoid switchbacks to the depot each time when a particular item has been picked, order pickers utilize devices like roll pallets or carts, which they pull or push along with them through the warehouse and on which they deposit the picked items until they finally return to the depot. Consequently, the required items are collected on tours through the warehouse, where the number of stops on each tour is limited by the available capacity of the picking device on the one hand and by the capacity requirements of the items to be picked on the other hand.

On their tours through the warehouse, order pickers are guided by pick lists. A pick list comprises a set of order lines, each one identifying a particular article, the quantity required of this article and the respective storage location. The order lines are already sorted into the sequence according to which the order picker is meant to collect the items. This sequence is usually determined by means of a so-called routing strategy, which can be seen as a heuristic targeted at the minimization of the length of the tour necessary to collect all items of the pick list.

2.2 Warehouse Layouts and Routing Strategies

Fig. 1a presents a *single-block layout* which is common for distribution warehouses. It consists of a number of (vertical) picking aisles arranged in parallel to each other and two (horizontal) cross aisles, one in the front and one in the back of the warehouse. The picking aisles between the two cross aisles establish a so-called block. Items are stored in and picked from racks on both sides of these picking aisles. Cross aisles do not contain any storage locations, but enable order pickers to move from one picking aisle to another. The depot is positioned at the center of the front cross aisle.

The picker route depicted in Fig. 1a is characterized by the fact that whenever a picking aisle has been entered (probably with the exception of the last aisle), it must be traversed completely. This establishes a routing scheme, in which the tours of the order pickers are built according to the *S-Shape* (or *Traversal*) strategy. From the depot the order picker proceeds to the leftmost aisle in which a requested item is located and traverses it entirely from the front to the back of the warehouse. Making use of the back cross aisle, the order picker then moves on to the next picking aisle which contains another item to be picked, and traverses that one from the back to

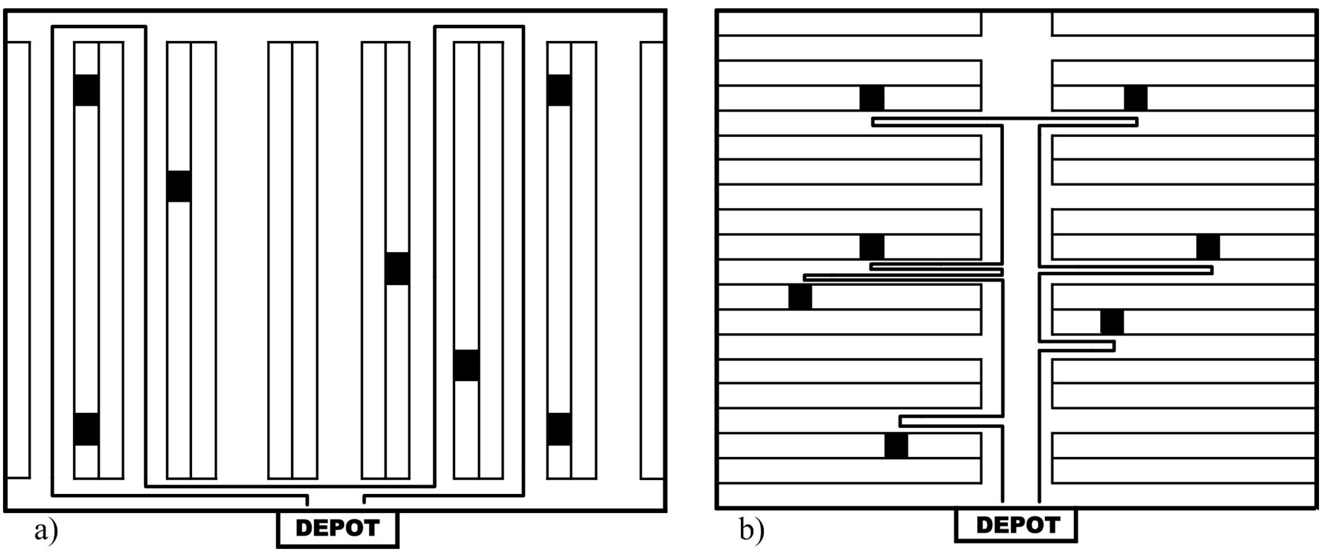

Figure 1: Traditional layouts of manual order picking systems: a) single-block and b) two-block layout

the front, etc. After having picked the last item indicated on the pick list, the order picker returns to the depot (Hall, 1993).

Fig. 1b depicts a *two-block layout* in which a (vertical) central aisle establishes two blocks of (horizontal) picking aisles; the picking aisles are arranged on both sides of the central aisle and run in parallel to the front of the warehouse. The depot is positioned in the middle of the front cross aisle, giving direct access to the central aisle (Bassan et al., 1980). The racks of the picking aisles are arranged flush with the (left and right) walls of the warehouse so that no further (vertical) cross aisles exist which would permit the order picker to cross over from one picking aisle to another.

As opposed to the layout of distribution warehouses, picking aisles are narrow so that picking devices cannot be maneuvered within, but are only operated in the central aisle. Starting at the depot, the order picker moves the picking device up to the first picking aisle of the left block from which an item has to be picked. The device is parked and the order picker enters the picking aisle without the device, picks the requested item and carries it back to the central aisle where the item is placed on the device. Any other item which might have to be picked from this aisle is treated in the same manner. Then the order picker continues the picking process by moving the picking device to the next picking aisle which contains a requested item. The items from this aisle are retrieved separately, again, etc. After having collected the items located in the aisle of the left block which is farthest from the depot, the picker switches to the block on the right and deals with the requested items located in the right block correspondingly on the way back to the depot. By doing so, a routing scheme is established which is based on the so-called *Return-with-Replication* strategy (Kunder and Gudehus, 1975).

Fig. 2 depicts a *U-shaped layout*, which has been proposed recently by Gerking (2009) for slow-moving item warehouses in particular. The central aisle is arranged in form of a U put upside down, i.e. the U consists of two vertical aisles (Vl and Vr) which are interconnected by a horizontal cross aisle (H). A front cross aisle (F) connects the central aisles to the depot. Central and front cross aisles are wide enough for allowing pickers to pass each other with their devices. However, in order to avoid congestions and blockings, in the central cross aisles which enclose the central block (shaded area in Fig. 2), all order pickers and their devices may only move in a single direction (clockwise or anti-clockwise). Furthermore, extensions (El and Er) of the vertical aisles exist, which run from the horizontal cross aisle (H) up to the northern wall of the warehouse. Unlike in the central aisle, here the picking devices may be maneuvered in both directions.

The central aisle and its extensions divide the picking area into four blocks: one central (horizontal) block below the U (1), one upper (vertical) block above the cross aisle (2) and one block of (horizontal) picking aisles each on the left (3l) and on the right (3r) of the U. The picking aisles can only be entered without the picking device, which is parked in the central aisles (or their extensions) while the requested items – one by one – are picked from their respective locations. Under these conditions, the so-called *Walking-the-U* strategy provides an appropriate routing scheme, which generally establishes picker routes according to the Return-with-Replication strategy. With respect to a requested item located in the central block, the order picker will enter the picking aisle either from the right or from the left central aisle, dependent on which one provides shorter access to the storage location. Different to the routes within two-block layouts, here pickers might have to alternate between picking from aisles on the left and aisles on right of the central aisle, since only one-way traffic is permitted in the central aisle. An example of a route resulting from this strategy is also depicted in Fig. 2.

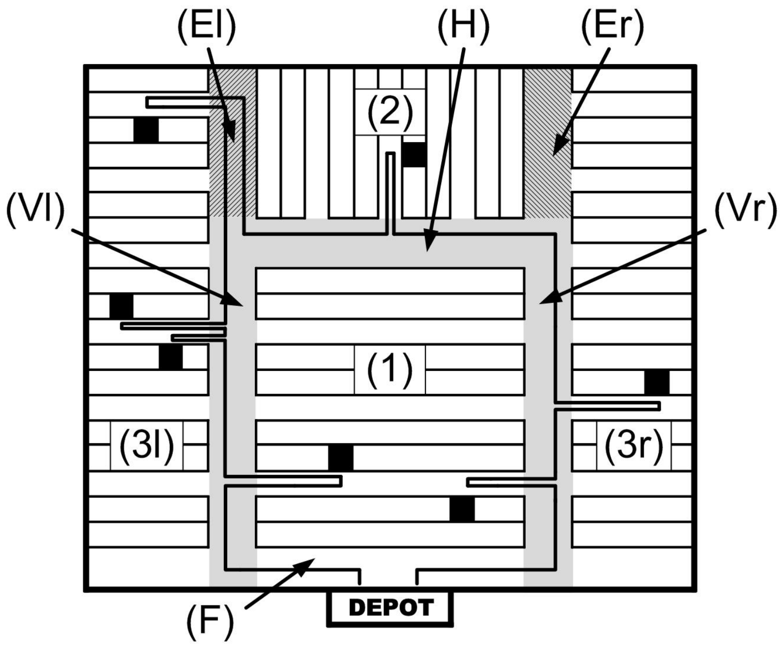

Figure 2: U-Shaped Layout

2.3 Space Requirements and Operational Safety

When designing an order picking warehouse for slow-moving items, efficient utilization of space is a central issue since the related costs (depreciation on land and buildings, (energy) costs of air conditioning and illumination) are crucial for the economic performance of the warehouse. The single-block layout of Fig. 1a does not appear to satisfy this requirement sufficiently. Due to the related routing strategy, the picking aisles have to be designed as wide aisles in order to allow for moving picking devices in both directions. This makes sense in distribution warehouses with a high turnover of the stored articles. For a warehouse for slow-moving items it results in a large amount of rarely used space.

The two-block layout of Fig. 1b requires much less space than the single-block layout does. The picking aisles can be kept narrow, since they must only allow for movements of order pickers but not for maneuvers of picking devices within. The central aisle, on the other hand, has to be designed as a wide aisle, because it must accommodate traffic of the picking devices in two opposite directions. Furthermore, picking devices will be parked here while the order pickers operate in the picking aisles. Thus, additional space must be allocated that is sufficient also for enabling pickers to pass such parked devices. However, the length of the central aisle is short in relation to the total length of the picking aisles. So in total, the space requirement of this layout is relatively small.

Due to the fact that traffic concentrates within the central aisle, where two-way traffic has to be accommodated, the two-block layout is prone to congestions, collision of picking devices and even accidents which might incur injuries of the pickers. The U-shaped layout of Fig. 2 avoids this drawback, since only one-way traffic is allowed in the central aisle, which generally reduces the probability of congestions and accidents. The picking aisles can be kept narrow again, while the central aisle, on the other hand, has to be relatively wide because – like in the two-block layout – parked picking devices will have to be passed, and, furthermore, pickers will have to cross – probably frequently – from one side of the central aisle to the other. Since the central aisle is relatively long in comparison to the one of the two-block layout, in total more space will be required for a U-shaped layout than for a two-block layout.

2.4 Average Tour Length Minimization

Apart from space requirements and safety issues described above, operating costs will determine the decision on the layout of an order picking warehouse. These costs consist of labor costs in the first place; thus, the time order pickers spend collecting the required items is another central aspect to be considered when deciding about the layout of an order picking warehouse. It is composed of the setup times of the tours, of the travel times (i.e. the times spent by the pickers for travelling to, from and between the locations of items to be picked), and of the actual retrieval times (i.e. the times for the identification and the picking of the items) (Tompkins et al., 2003). Among these components, the travel time consumes the largest proportion of the total order picking time. The other components, i.e. setup time and retrieval time, can be looked upon as constants. Thus, assuming the picker's travel velocity to be constant, the minimization of the (average) travel time per picking order is equivalent to the minimization of the (average) picking tour length (Jarvis and McDowell, 1991).

Which of the above-described layouts results in shorter tour lengths cannot be answered directly. The number of items to be picked on a tour (picks per tour) and the demand frequency of the item types (i.e. the number of times an item type appears in a customer order during a specific time period), will have a significant effect on the relative superiority of one layout (including its corresponding routing strategy) over another. Therefore, we would like to demonstrate under which conditions one layout type outperforms the others with respect to the average tour length.

3 Literature Review

The layout of order picking systems and the estimation of travel times have been frequently discussed in the literature on distribution warehouses. The majority of the publications focus on single- or multi-block layouts, where picking aisles are located in parallel to each other. Evaluation of the performance of a layout and its respective routing scheme is typically performed in two different ways. The first one consists of the development of analytical expressions for the expected value of the tour length per picking order, while the second one uses simulation, i.e. repeated numerical experiments, for the determination of the average tour length.

An early contribution of assessing the impact of layout parameters (aisle length, aisle width, and number of storage locations) on the average travel distances in a single-block layout has been presented by Kunder and Gudehus (1975). Assuming uniformly distributed demands (i.e. the demand frequencies of the item types are identical), the authors estimate the expected tour lengths when different routing strategies (e.g. S-Shape routing and Return-with-Replication routing) are applied. The expected values depend on the above-mentioned layout parameters and the number of picks per tour. Bassan et al. (1980) propose decision rules for the determination of layout parameters in a single- and two-block layout based on the annual article throughput, the costs per unit area of the warehouse and material handling costs. Hall (1993) also considers a single-block layout where the demands are uniformly distributed. For four routing strategies the expected tour lengths are given and it is demonstrated that they are significantly dependent on the number of aisles and the number of picks per tour. Hwang et al. (2004) compare the performance of three routing strategies in a single-block layout where class-based demands are assumed. They show that whether and to what extent one strategy is superior to another depends on the size of the order picking area, the demand frequencies and the number of picks per tour. Caron et al. (2000) present an estimation of the expected tour length obtained by S-Shape routing in a two-block layout. For class-based demands and varying the number of picks per tour the authors determine the number of aisles which minimizes the expected tour length. Similarly, for a single-block layout with a given number of storage locations and S-Shape routing, Roodbergen and Vis (2006) determine the number of aisles for which the expected tour length is minimal.

For a multi block layout with S-Shape routing, Roodbergen et al. (2008) determine the number of blocks and aisles which provide the shortest tour length. Their results are based on tour length estimations and simulation, assuming uniformly distributed demands. Roodbergen and de Koster (2001) identify the best routing strategy for a given multi block layout by simulation.

Very few approaches only deal with non-standard warehouse layouts, where central and picking aisles are not necessarily arranged orthogonally to each other. White (1972) considers a radial aisle structure, in which aisles project away from the depot. Assuming that the aisles have a width of zero, he determines the number of aisles which minimizes the sum of the euclidean distances between depot and storage locations. Gue and Meller (2009) and Pohl et al. (2009) analyze two different layout types each. The first type is a two-block layout with a curved cross aisle instead of a straight one (called flying-V layout). The second type consists of a (straight) diagonal cross aisle with vertical picking aisles above and horizontal picking aisles below (called fishbone layout). For the flying-V layout Gue and Meller (2009) establish the position and the slope of the cross aisle with respect to a minimization of the expected travel distance from the storage locations to the depot. They show – in comparison to a corresponding single-block layout – that the flying-V and the fishbone layout may reduce the expected tour length by 10% and 20%. Pohl et al. (2009) analyze the expected tour length for the fishbone layout in a dual-command warehouse. The authors demonstrate that the tour length can be improved by 10% to 15% in comparison to the ones in warehouses with traditional layouts.

4 Expected Tour Lengths

4.1 General Notation

With respect to the estimation of the expected tour length of a picking order we will use the following constants, independently from the specific layout and the corresponding routing strategy:

k: total number of items to be picked on a tour;
n^{loc}: total number of storage locations in the warehouse;
p_i^{loc}: probability that an item has to be picked from storage location i ($i \in \{1, \ldots, n^{loc}\}$).

As indicated in Fig. 1 and 2, it is assumed that the depot is positioned at the center of the front of the warehouse.

4.2 Single-Block Layout with S-Shape Routing

Chew and Tang (1999) determine the expected tour length for a single-block layout in which picking tours are formed according to the S-Shape routing scheme (in the following the superscript S indicates that the respective symbols are related to this particular combination of warehouse layout and routing scheme). For the presentation of this model, the following notation is introduced:

m^S: number of picking aisles; the aisles are numbered consecutively from the left to the right in such a way that the index 1 is assigned to the leftmost aisle and the index m^S to the rightmost aisle;
d_a^S: total length of each picking aisle;
d_c^S: center-to-center distance between two adjacent picking aisles;
$\mathcal{I}_r$: set of all storage locations in picking aisle r ($r \in \{1, \ldots, m^S\}$);
p_r^a: probability that at least one item has to be picked from a storage location in picking aisle r ($r \in \{1, \ldots, m^S\}$); $p_r^a = 1 - \prod_{i \in \mathcal{I}_r}(1 - p_i^{loc})$;
J: number of picking aisles which have to be visited;
G_l: distance (defined in the number of picking aisles) to be travelled by the order picker in the cross aisles when moving into the left direction from the depot;
G_r: distance (defined in the number of picking aisles) to be travelled by the order picker in the cross aisles when moving into the right direction from the depot;
TL^S: tour length provided by S-Shape routing;
$E[TL^S|k]$: expected tour length provided by S-Shape routing if k items have to be picked;
$E[J|k]$: expected number of visited picking aisles if k items have to be picked;
$E[G_l|k]$: expected distance (defined in the number of picking aisles) to be travelled by the order picker in the cross aisles when moving into the left direction from the depot if k items have to be picked;
$E[G_r|k]$: expected distance (defined in the number of picking aisles) to be travelled by the order picker in the cross aisles when moving into the right direction from the depot if k items have to be picked.

In a single-block layout with S-Shape routing, the expected length of a tour on which k items have to be picked is then given by

$$E[TL^S|k] = d_a^S E[J|k] + 2d_c^S E[G_l|k] + 2d_c^S E[G_r|k] + d_a^S P(J \text{ is odd}|k). \quad (1)$$

The first component of this sum, $d_a^S E[J|k]$, expresses the expected distance to be travelled by the order picker within the picking aisles in order to collect the requested items, i.e. the length of a picking aisle multiplied by the expected number of aisles to be visited (and to be traversed completely). The sum of the second and third component, i.e. $2d_c^S E[G_l|k]+2d_c^S E[G_r|k]$, represents the travel distance in the cross aisles, consisting of the center-to-center distance between two adjacent aisles multiplied by the number of aisles the order picker is expected to be travelling from the depot to the left and to the right in the front and the back cross aisle, respectively. The last component, $d_a^S P(J \text{ is odd}|k)$ determines the additional travel distance if an odd number of aisles has to be visited. According to Chew and Tang (1999) the four components can be computed as follows:

$$E[J|k] = m^S - \sum_{j=1}^{m^S} (1 - p_r^a)^k; \tag{2}$$

$$E[G_l|k] = \max\{0, m^S/2 - \sum_{j=1}^{m^S/2-1} (\sum_{r=1}^{j} p_r^a)^k\}; \tag{3}$$

$$E[G_r|k] = \max\{0, m^S/2 - \sum_{j=1}^{m^S/2-1} (\sum_{r=1}^{j} p_{m^S-r}^a)^k\}; \tag{4}$$

$$P(J \text{ is odd}|k) = 1/2 + 1/2 \sum_{j=1}^{m^S} (\sum_{i}^{(j)} (-1)^{j-1} (\sum_{r=1}^{j} p_{i_r}^a)^k 2^{m^S-1}), \tag{5}$$

where $\sum_i^{(j)}$ represents the summation over all subsets $i = (i_1, i_2, \ldots, i_j)$ of j integers from m^S integers.

4.3 Two-Block Layout with Return-with-Replication Routing

With respect to tour length estimation for a two-block layout in which picking tours are generated by the Return-with-Replication routing scheme (indicated by the superscript R) we introduce the following notation:

m^R:	number of picking aisles, i.e. $m^R/2$ picking aisles each on both sides of the central aisle; the aisles on the left are numbered consecutively from the front to the back of the warehouse in such a way that the index 1 is assigned to the aisle next to the depot and the index $m^R/2$ to the aisle located farthest from the depot; the aisles on the right from the central aisle are numbered analogously, i.e. the index $m^R/2+1$ is assigned to the aisle next to the depot and the index m^R to the aisle located farthest from the depot;
d_c^R:	center-to-center distance between two adjacent picking aisles;
$\mathcal{I}_r$:	set of storage locations in picking aisle r ($r \in \{1, \ldots, m^R\}$);
d_i:	distance of storage location i ($i \in \mathcal{I}_r, r \in \{1, \ldots, m^R\}$) from the cross aisle;
p_r^a:	probability that at least one item has to be picked in picking aisle r and the picking aisle located across of the central aisle, i.e. $r + m^R/2$ ($r \in \{1, \ldots, m^R/2\}$), $p_r^a = 1 - \prod_{i \in I_r} (1 - p_i^{loc}) \prod_{i \in I_{m^R/2+r}} (1 - p_i^{loc})$;
G:	index of the visited aisle farthest from the depot ("farthest visited aisle");
$E[G\|k]$:	expected index of the farthest visited aisle if k items have to be picked;
TL_p:	travel distance within the picking aisles;
$E[TL_p\|k]$:	expected travel distance within the picking aisles if k items have to be picked;
TL^R:	tour length provided by Return-with-Replication routing for the two-block layout;
$E[TL^R\|k]$:	expected tour length provided by Return-with-Replication routing for the two-block layout if k items have to be picked.

The length of a tour to be travelled by an order picker is made up of the travel distance within the picking aisles and the travel distance within the central aisle. Therefore, the expected tour length $E[TL^R|k]$ of an order that requires k items to be picked, can be determined as

$$E[TL^R|k] = E[TL_p|k] + 2d_c^R E[G|k]. \tag{6}$$

We note that the order picker has to enter each aisle separately for each requested item. Under the assumption that the appearance of an item in a picking order is independent from the appearance of other items in the order, the expected travel distance within the picking aisles $E[TL_p|k]$ can be computed as

$$E[TL_p|k] = k \sum_{r=1}^{m^R} \sum_{i \in \mathcal{I}_r} 2d_i p_i^{loc}. \tag{7}$$

For the determination of the expected travel distance within the cross aisle $E[G|k]$, the expression derived in Subsection 4.2 can be adapted, and one obtains

$$E[G|k] = m^R/2 - \sum_{j=1}^{m^R/2-1} (\sum_{r=1}^{j} p_r^a)^k - m^R/4 = m^R/4 - \sum_{j=1}^{m^R/2-1} (\sum_{r=1}^{j} p_r^a)^k. \tag{8}$$

4.4 U-Shaped Layout with Walking-the-U Routing

In order to estimate the expected tour length for a picking order in a U-shaped layout with Walking-the-U routing (U) we introduce the following notation (also see Fig. 2 for the notation of blocks and aisles):

m_1^U: number of picking aisles in the central block (1);
m_2^U: number of picking aisles in the upper block (2);
m_{3l}^U: number of picking aisles in the left block (3l);
m_{3r}^U: number of picking aisles in the right block ($m_{3l}^U = m_{3r}^U$) (3r);
$\mathcal{I}_r^1$: set of all storage locations in a picking aisle r in block (1) ($r \in \{1, \ldots, m_1^U\}$);
$\mathcal{I}_r^2$: set of all storage locations in a picking aisle r in block (2) ($r \in \{1, \ldots, m_2^U\}$);
$\mathcal{I}_r^{3l}$: set of all storage locations in a picking aisle r in block (3l) ($r \in \{1, \ldots, m_{3l}^U\}$);
$\mathcal{I}_r^{3r}$: set of all storage locations in a picking aisle r in block (3r) ($r \in \{1, \ldots, m_{3r}^U\}$);
$d'_{i/r}$: distance from storage location i or picking aisle r to the central aisle;
TL_c: travel distance in the central and front aisle (F, Vr, H, Vl);
TL_p: travel distance within the picking aisles;
$E[TL_p|k]$: expected travel distance within the picking aisles if the picking order requires k items to be picked;
TL_{El}: travel distance in the left extension (El) of the central aisle;
TL_{Er}: travel distance in the right extension (Er) of the central aisle;
$E[TL_{El}|k]$: expected travel distance in El if k items have to be picked;
$E[TL_{Er}|k]$: expected travel distance in Er if k items have to be picked;
TL^U: tour length provided by Walking-the-U routing in the U-shaped layout;
$E[TL^U|k]$: expected tour length provided by Walking-the-U routing in the U-shaped layout if k items have to be picked.

The expected length of a picker tour is composed of four components, the (fixed) travel distance in the central and in the front aisle (F, Vr, H, Vl), TL_c, the expected travel distance within the picking aisles, $E[TL_p|k]$, the expected travel distance in the left extension (El), $E[TL_{El}|k]$, and the expected travel distance in the right extension (Er), $E[TL_{Er}|k]$

$$E[TL^U|k] = TL_c + E[TL_p|k] + E[TL_{El}|k] + E[TL_{Er}|k]. \tag{9}$$

In analogy to the expression derived for described routing schemes, the expected travel distance in the picking aisles can be calculated here as follows:

$$E[TL_p|k] = k(\sum_{r=1}^{m_1^U}\sum_{i\in\mathcal{I}_r^1} 2d_ip_i(k) + \sum_{r=1}^{m_2^U}\sum_{i\in\mathcal{I}_r^2} 2d_ip_i(k) + \sum_{r=1}^{m_{3l}^U}\sum_{i\in\mathcal{I}_r^{3l}} 2d_ip_i(k))\sum_{r=1}^{m_{3r}^U}\sum_{i\in\mathcal{I}_r^{3r}} 2d_ip_i(k). \tag{10}$$

With respect to the estimation of $E[TL_{El}|k]$, i.e. the expected travel distance in (El), the far most pick location can either be located in an aisle of the left block or in the first aisle of the upper block. Therefore, $E[TL_{El}|k]$ can be expressed as follows:

$$\begin{aligned}
E[TL_{El}|k] = &\sum_{i\in I_1^2} 2d_i'P(\text{at least one pick in } i) \prod_{i\in I_1^2, i'>i} P(\text{no pick in } i') \prod_{r\in\{m_2^U,\dots,m_3^{Ul}\}|d_r'>d_i} P(\text{no pick in } r)\\
&+ \sum_{r=m_1^U}^{m_{3l}^U} 2d_r'P(\text{at least one pick in } r)\cdot \prod_{r'=r+1}^{m_{3l}^U} P(\text{no pick in } r') \prod_{i\in I_1^2|d_i'>d_r'} P(\text{no pick in } i)\\
= &\sum_{i\in I_1^2} 2d_i'(1-(1-p_i^{loc})^k) \prod_{i\in I_1^2, i'>i} (1-p_{i'}^{loc})^k \prod_{r\in\{m_1^U,\dots,m_3^{Ul}\}|d_r'>d_i} \prod_{i'\in I_r^{3l}} (1-p_{i'}^{loc})^k\\
&+ \sum_{r=m_1^U}^{m_{3l}^U} 2d_r'(1-\prod_{i\in I_r^{3l}}(1-p_i^{loc})^k) \prod_{r'=r+1}^{m_{3l}^U}\prod_{i'\in I_{r'}^{3l}} (1-p_{i'}^{loc})^k \prod_{i'\in I_1^2|d_i'>d_r'} (1-p_{i'}^{loc})^k
\end{aligned}$$

Likewise, the expected travel distance $E[TL_{Er}|k]$ in (Er) can be derived as

$$\begin{aligned}
E[TL_{Er}|k] = &\sum_{i\in I^2_{m^2_U}} 2d_i'(1-(1-p_i^{loc})^k) \prod_{i\in I^2_{m^2_U}, i'>i} (1-p_{i'}^{loc})^k \prod_{r\in\{m_2^U,\dots,m_{3r}^U\}|d_r'>d_i} \prod_{i'\in I_r^{3r}} (1-p_{i'}^{loc})^k\\
&+ \sum_{r=m_1^U}^{m_{3r}^U} 2d_r'(1-\prod_{i\in I_r^{3r}}(1-p_i^{loc})^k) \prod_{r'=r+1}^{m_{3r}^U}\prod_{i'\in I_{r'}^{3r}} (1-p_{i'}^{loc})^k \prod_{i'\in I^2_{m_2^U}|d_i'>d_r'} (1-p_{i'}^{loc})^k.
\end{aligned}$$

5 Verification of Estimations

By means of numerical experiments, we will verify the analytical expressions derived for the expected tour lengths in Section 4.

5.1 Problem Parameters

In the experiments, we consider a picking area, where 1,600 articles are to be stored in total. However, due to the specific dimensions and layout parameters of each layout type, it will not be possible to include exactly 1,600 storage locations in the picking area of the single- and two-block layout. In such case, a minimal number of storage locations has been added. These additional locations have been assigned to places located farthest from the depot and no articles have been allocated to them. Each storage location has a length of 0.3m and a width of 0.6m. Table 1 gives an overview over the warehouse dimensions related to the three layout types. As has been mentioned in Section 2, the single-block layout requires significantly more space than the two block layout or the U-shaped layout.

For the number of picks per tour (k) we consider eight different values (5, 10, 15, 20, 25, 30, 35, 40). Correspondingly, we obtain pick densities (i.e. number of picks over the number of storage locations to be visited on a picking tour) of 0.003125, 0.00625, 0.009375, 0.0125, 0.015625, 0.01875, 0.021875, and 0.025.

With respect to the demands of the articles we assume two different types of distributions: (a) Uniformly distributed demands, i.e. the probability that a particular article has to be picked amounts to 1/1600. According to this assumption, the articles have been assigned randomly to storage locations of the respective layout. (b) Class-based demands, i.e. according to their demand frequencies, the articles are grouped into three classes, A, B and C, where A contains articles of high, B of medium-size and C of low demand. More precisely, A includes articles representing 80% of the demand, while the articles in B and C make up for 15% and 5% of the demand, respectively. In the single-block layout the articles of class A have been assigned to the aisles next to the depot. Of the aisles being available after this assignment, the ones located closest to the depot have been taken for the accommodation of the articles of class B. Articles of class C have been assigned to the remaining aisles. Within the (set of) aisles assigned to a particular article class, the articles have been assigned randomly to storage locations. In the two-block layout and in the U-shaped layout the articles have been assigned to storage locations according to the distance from the respective location to the central aisle, i.e. articles of class A are stored at the locations next to the central aisle. From those locations not yet allocated, the ones next to the central aisles are taken in order to accommodate articles of class B, while the locations remaining after this assignment accommodate articles of class C. Again, within the set of locations to which a particular class of articles has been assigned, a random storage policy has been applied.

Combination of the parameter values provides 136 problem classes in total. For each problem class, 10,000 problem instances have been generated randomly.

parameter	**single-block layout**	**two-block layout**	**U-shaped layout**
total size	937 m^2	505 m^2	666 m^2
total length	24.4 m	23.4 m	22.2 m
total width	38.4 m	21.6 m	30.0 m
number of aisles	$m^S = 12$	$m^R = 24$	$m_1^U = 6$; $m_2^U = 7$ $m_3^{Ul} = m_3^{Ur} = 10$
storage locations per aisle	2 × 68	2 × 34	40 (block (1)) 20 ((2), (3l), (3r))
total number of locations	1,623	1,632	1,600
width of central aisle	2 m	3 m	3 m
width of picking aisle	2 m	0.6 m	0.6 m
center-to-center distance	$d_c^S = 3.2$ m	$d_c^R = 1.8$ m	
length of a picking aisle	$d_a^S = 20.4$ m	10.2 m	

Table 1: Layout parameters for single-block, two-block and U-shaped layout

5.2 Comparison of Expected Tour Lengths and Average Tour Lengths

In order to verify our findings, for each problem class and layout type the corresponding tour length has been determined in two ways, namely (1) according to the expressions derived for the expected tour length in Section 4, and (2) as the average tour length calculated as the algebraic mean of the actual tour lengths of the instances in a class. By comparing the expected tour length and the average tour length the quality of our approximation will be demonstrated.

Tables 6 - 8 give an overview of the deviations of the expected tour lengths (computed on the basis of the derived expressions) from the average tour lengths (obtained from the numerical experiments) for the three combinations of layout types and routing strategies.

For every problem class it can be noted that the expected tour length is very close to the average tour length obtained from numerical experiments. In general, the expected tour length (for class-based demands) slightly overestimates the actual average tour length; however, the deviations are rather small. The largest deviations can be observed for the single-block layout, where an average deviation of up to 1.0% can be noticed and the maximal deviation amounts to 1.7%. For the two-block layout and the U-shaped layout the average deviations are at most 0.5% and 0.8%, respectively, while the corresponding maximal deviations account for 1.2% and 2.3%.

In a similar study Roodbergen et al. (2008) determine the number of blocks and aisles providing the shortest tour lengths for commonly used routing strategies. In their experiments they observed an average deviation between tour lengths obtained from analytic analysis and numerical experiments of 2.15% while the corresponding maximal deviation rise to 3.7% which they considered as acceptable. Consequently, we conclude that our findings have been satisfactorily verified by the numerical experiments.

demand			**picks per tour k**							
			5	10	15	20	25	30	35	40
uniformly distributed			-0.2	-0.3	-0.2	-0.1	-0.1	-0.1	-0.1	0.0
class-based										
quan. A	quan. B	quan. C								
45%	40%	15%	0.4	0.8	0.6	0.5	0.9	1.0	0.7	0.8
35%	35%	30%	0.3	0.4	0.6	0.6	0.7	0.7	0.6	0.7
30%	35%	35%	0.0	0.8	0.5	0.7	0.6	0.1	0.4	0.7
20%	35%	45%	0.5	0.8	0.6	0.8	0.9	0.8	1.2	0.5
20%	25%	55%	0.4	0.5	1.0	1.0	0.6	0.7	0.5	0.6
15%	35%	50%	0.4	0.9	0.2	0.7	0.2	0.8	0.7	0.6
15%	25%	60%	0.1	0.1	0.6	0.8	0.9	0.5	0.9	0.7
10%	45%	45%	0.3	1.4	1.7	1.4	1.2	1.8	1.1	0.8
10%	35%	55%	0.5	0.8	1.0	0.8	1.5	1.0	1.6	1.0
10%	25%	65%	0.5	0.8	1.3	1.1	1.2	1.0	1.3	1.4
10%	15%	75%	0.3	1.4	1.1	1.4	1.4	0.9	1.3	0.9
5%	55%	40%	0.8	1.1	1.6	1.5	0.6	1.3	1.1	1.3
5%	45%	50%	0.6	1.0	0.8	0.9	1.3	1.1	1.0	1.3
5%	35%	60%	0.6	0.8	0.9	1.1	1.3	1.2	1.0	1.0
5%	25%	70%	0.5	1.1	0.8	1.1	1.3	1.0	1.4	1.0
5%	15%	80%	0.2	0.3	1.1	1.3	1.4	1.5	1.6	1.1
average			0.4	0.8	0.8	0.9	0.9	0.9	1.0	0.8

Table 2: Deviation [in %] of the expected tour length from the (average) tour length obtained by simulation in the single-block layout

demand			**picks per tour** k							
			5	10	15	20	25	30	35	40
uniformly distributed			-0.2	-0.5	-0.4	-0.2	-0.2	-0.4	-0.4	0.3
class-based										
quan. A	quan. B	quan. C								
45%	40%	15%	0.2	0.3	0.3	0.2	0.2	0.4	0.4	0.4
35%	35%	30%	0.5	0.1	0.2	0.1	0.2	0.2	0.2	0.2
30%	35%	35%	0.4	0.4	0.1	0.5	0.4	0.3	0.4	0.4
20%	35%	45%	0.3	0.5	0.6	0.3	0.5	0.5	0.5	0.2
20%	25%	55%	0.3	0.4	0.4	0.3	0.3	0.3	0.5	0.5
15%	35%	50%	0.3	0.4	0.7	0.4	0.6	0.6	0.6	0.7
15%	25%	60%	0.2	0.6	0.6	0.3	0.4	0.4	0.5	0.6
10%	45%	45%	0.4	0.8	0.7	0.6	0.5	0.7	0.3	0.4
10%	35%	55%	0.4	0.5	0.6	0.3	0.4	0.2	0.6	0.4
10%	25%	65%	0.5	0.6	0.7	0.4	0.3	0.3	0.5	0.5
10%	15%	75%	0.0	0.9	0.5	0.5	0.6	0.4	0.5	0.2
5%	55%	40%	0.6	0.7	0.8	0.6	0.4	0.3	0.3	0.4
5%	45%	50%	0.5	0.8	0.4	0.3	0.5	0.6	0.6	0.6
5%	35%	60%	1.2	0.8	0.5	0.5	0.5	0.5	0.5	0.5
5%	25%	70%	0.0	0.5	0.5	0.5	0.7	0.5	0.6	0.4
5%	15%	80%	0.3	0.3	0.5	0.5	0.5	0.4	0.5	0.3
average			0.4	0.5	0.4	0.4	0.4	0.4	0.4	0.4

Table 3: Deviation [in %] of the expected tour length from the (average) tour length obtained by simulation in the two-block layout

demand			**picks per tour** k							
			5	10	15	20	25	30	35	40
uniformly distributed			2.3	2.2	2.2	1.9	1.4	1.1	1.0	0.7
class-based										
quan. A	quan. B	quan. C								
45%	40%	15%	1.3	1.5	1.6	1.6	1.3	1.2	1.1	0.8
35%	35%	30%	1.0	1.3	1.5	1.4	1.3	1.1	1.0	1.0
30%	35%	35%	0.7	0.9	1.0	0.9	0.9	1.1	0.9	0.7
20%	35%	45%	0.7	1.0	1.1	1.2	1.1	1.1	0.9	1.1
20%	25%	55%	0.8	1.1	1.1	1.2	1.2	1.2	1.1	1.0
15%	35%	50%	0.6	0.9	0.8	0.9	0.8	0.9	0.9	0.8
15%	25%	60%	0.6	0.6	0.9	1.0	1.1	0.9	0.9	0.8
10%	45%	45%	0.3	0.2	0.1	0.3	0.3	0.1	0.4	0.5
10%	35%	55%	0.2	0.2	0.3	0.4	0.2	0.4	0.3	0.4
10%	25%	65%	0.1	0.1	0.1	0.4	0.3	0.4	0.2	0.3
10%	15%	75%	0.2	0.1	0.1	0.1	0.0	0.3	0.2	0.3
5%	55%	40%	0.4	0.5	0.6	0.5	0.9	0.7	0.8	0.6
5%	45%	50%	0.4	0.4	0.7	0.7	0.5	0.6	0.6	0.5
5%	35%	60%	0.2	0.3	0.6	0.5	0.6	0.5	0.7	0.7
5%	25%	70%	0.4	0.4	0.5	0.5	0.4	0.6	0.5	0.6
5%	15%	80%	0.3	0.5	0.5	0.5	0.4	0.5	0.5	0.6
average			0.6	0.7	0.8	0.8	0.8	0.8	0.7	0.7

Table 4: Deviation [in %] of the expected tour length from the (average) tour length obtained by simulation in the U-shaped layout

6 Analysis of Estimations

In this section, we will use the derived analytic expressions in order to identify under which conditions and to what extent one layout type – in particular the newly introduced U-shaped layout – outperforms the others. In order to do so, we will refer to the same problem classes which have been used in the previously described numerical experiments.

6.1 Expected Tour Lengths for Uniformly Distributed Demands

The entries of Table 5 present the layout types which – in combination with the respective routing schemes – provide the shortest expected tour length for uniformly distributed demands with respect to a different number of picks per tour. For a small number of picks ($k = 5$), the two-block layout is superior to the other layout types, while the single-block layout turns out to be the best for a large number of picks ($k = 30; 35; 40$). For a medium-sized number of picks ($k = 10; 15; 20; 25$), the U-shaped layout outperforms the other two layout types. This already demonstrates that – along with the single- and two-block layouts – the U-shaped layout must be considered as another relevant option when warehouses for slow-moving items are to be designed.

picks per tour k	5	10	15	20	25	30	35	40
uniformly distributed demands	R	U	U	U	U	S	S	S

Table 5: Layout type which provides the shortest expected tour length for uniformly distributed demands ('S': single-block layout with S-Shape routing; 'R': two-block layout with Return-with-Replication routing; 'U': U-shaped layout with Walking-the-U routing)

Table 6 depicts how much the expected tour length of a given layout type deviates from the expected tour length of the corresponding best layout type. It shows that choosing the "wrong" layout type may result in picker tours significantly longer than those related to the best layout type. The single-block layout is obviously inappropriate for a small number of picks per tour, where – for $k = 5$ – the deviation amounts to 64%. The deviation decreases when the number of picks per tour grows. In contrast to that, the two-block layout represents the best layout type for $k = 5$, but is inferior to the other two layout types for all larger numbers of picks ($k \geq 10$). The deviation becomes more significant as the number of picks grows and it reaches a maximum of 55% (for $k = 40$).

picks per tour k	5	10	15	20	25	30	35	40
single-block layout	63.7	69.5	36.9	23.0	10.2	0.0	0.0	0.0
two-block layout	0.0	4.0	10.8	15.3	18.5	22.5	38.5	55.3
U-shaped layout	10.6	0.0	0.0	0.0	0.0	1.2	12.6	24.6

Table 6: Deviation [in %] of the expected tour length provided by the layout types under discussion from the respective shortest expected tour length (uniformly distributed demands)

The increasing inferiority of the two-block layout against the single-block layout for a larger number of picks per tour can be explained on the basis of the underlying routing strategies. We remark that the number of items to be picked from a single aisle grows as the number of picks per tour grows. According to the S-Shape strategy, which is applied in connection with the single-block layout, the order picker has to enter an aisle just once independently from the number of items to be collected from this aisle, while the Return-with-Replication strategy used for the two-block layout necessitates the order picker to collect each item separately and to enter an aisle several times which results in longer tours.

On the other hand, for a very low number of picks per tour ($k = 5$) it is unlikely that more than a single item has to be collected from a picking aisle. Thus, for the single-block layout the above-mentioned advantages of the S-Shape routing strategy cannot become effective. Furthermore, in the two-block layout access to an item to be picked requires only a relatively short distance to be travelled from the central aisle since each picking aisles is (more or less) only half as long as a picking aisles of the single-block layout. Since the S-Shape routing strategy enforces that the order picker always traverses a picking aisle completely, the expected total distance to be travelled within the picking aisles must be smaller in case of the two-block layout than in case of the single-block layout. Furthermore, additional advantages of the two-block layout stem from the fact that the picking aisles are narrow. Thus, also the distances to be travelled in the central aisle must be shorter than those to be travelled in the cross aisles of the single-block layout.

The U-shaped layout is inferior to the two-block layout for $k = 5$. Similar to the two-block layout, it consists of a larger number of (shorter) picking aisles than the single-block layout does. However, the advantages stemming from the corresponding reduction of the required distance to be travelled within the picking aisles is partially compensated by an additional distance to be travelled within the central aisle, which is longer than the total length of the cross aisles of the two-block layout. In fact, the number of picking aisles which can be accessed according to the Return-with-Replication strategy is even larger in the U-shaped layout than in the two-block layout. Therefore, already for $k = 10$, the reduction of the travel distance within the picking aisles is no longer overcompensated by additional travel within the central aisle, and the U-shaped layout becomes the layout type which is superior to the other two layout types. The advantages of the S-Shape strategy only have a clear impact for a relatively large number of picks per tour ($k \geq 30$), when the single-block layout outperforms the U-shaped layout.

We conclude that the U-shaped layout is definitely a layout type to be considered when the demands are uniformly distributed and a medium-sized number of items has to be picked per tour. We further note that the U-shaped layout also represents a relatively insensitive solution in the sense that even if the actual number of picks per tour is outside the "optimal" range of this layout type, i.e. in case that k is smaller than 10 or slightly larger than 25, then the deviation of the expected tour length for the U-shaped layout from the corresponding expected tour length of the respective best layout is still small and acceptable (11% for $k = 5$, and 1% and 13% for $k = 30$ and $k = 35$, respectively).

6.2 Expected Tour Lengths for Class-Based Demands

Differentiated again with respect to the number of picks per tour, Table 7 depicts which layout type performs best for class-based demands. For different assumptions concerning the proportions according to which the 1,600 articles are assigned to the three classes A, B, and C, and the layout types are presented which provide the shortest expected tour lengths. The row on the bottom of the table (# 16) represents the most skewed (unbalanced) demand distribution: 5% of the items make up for 80% of the demand (class A), another 15% of the articles account for 15% of the demand (class B), and the final 80% of the items include 5% of the demand (class C; also see Section 5.1).

demand				picks per tour k							
#	quan. A	quan. B	quan. C	5	10	15	20	25	30	35	40
1	45%	40%	15%	R	U	U	U	U	U	S	S
2	35%	35%	30%	R	U	U	U	U	U	S	S
3	30%	35%	35%	R	U	U	U	U	U	S	S
4	20%	35%	45%	R	U	U	U	U	U	S	S
5	20%	25%	55%	R	U	U	U	U	S	S	S
6	15%	35%	50%	R	U	U	U	U	S	S	S
7	15%	25%	60%	R	U	U	U	S	S	S	S
8	10%	45%	45%	R	R	U	U	U	U	S	S
9	10%	35%	55%	R	R	U	U	U	S	S	S
10	10%	25%	65%	R	R	U	U	S	S	S	S
11	10%	15%	75%	R	R	U	S	S	S	S	S
12	5%	55%	40%	R	R	U	U	U	U	S	S
13	5%	45%	50%	R	R	U	U	U	U	S	S
14	5%	35%	60%	R	R	U	U	U	S	S	S
15	5%	25%	70%	R	R	U	S	S	S	S	S
16	5%	15%	80%	R	R	S	S	S	S	S	S

Table 7: Layout type which provides the smallest expected tour length for class-based demands

Independently from the specific demand distribution it can be observed that the two-block layout outperforms the other layout types for a small number of picks per tour ($k = 5$), while the single-block layout is superior for a large number ($k = 35; 40$). As has been described for the uniformly distributed demands, the corresponding routing strategies fit these situations particularly well. A medium-sized number of picks (k = 10; 15; 20; 25; 30) again characterizes the set of problem classes where an implementation of the U-shaped layout tends to be favorable. However, in contrast to the case of uniformly distributed demands, this can only be considered a tendency here but is not true for all demand distributions. Particularly noticeable are the entries for the most unbalanced distribution in the last row (# 16) of Table 7. In the single- and two-block layout, the items of class A will be found in the picking aisles next to the depot. The other articles are located further away from the depot; however, only in rare cases tours will lead pickers to their locations since these items have a low demand. In other words, in the majority of problem instances, tours will be concentrated in picking aisles next to the depot. Thus, for a small number of picks per tour (k = 5; 10), the two-block layout in combination with the Return-with-Replication routing strategy outperforms the combination of the single-block layout and the S-Shape strategy. On the other hand, for a larger number of picks ($k \geq 15$) the latter combination is superior to the first one. The U-shaped layout cannot become advantageous at all because the (long) central aisle has to be travelled totally on each tour. The same effects can also be observed for other problem classes, though to a smaller extent. They explain, in particular, the superiority of the two-block layout for $k = 10$ and a strong concentration of demands in class A (proportion of articles in class A: 5%; 10%; rows # 8 - 16), and likewise the superiority of the single-block layout for medium-sized number of picks, when the proportion of articles in class C is relatively high (rows # 5, 6, 7, 9, 10, 11, 14, and 15).

Tables 8, 9, and 10 depict how much the expected tour length of a given layout type deviates from the expected tour length of the corresponding best layout type. It becomes clear again that a changeover from the respective best layout type may result in significant additional distances to be travelled by the order picker. For the single-block layout (cf. Table 8), the respective expected tour length may be exceeded by up to 78% ($k = 5$, demand distributions of rows # 13, and 14). With respect to 11 (out of 128) problem classes the deviation ranges between 50 and 75%. For

#	demand quan. A	quan. B	quan. C	picks per tour k 5	10	15	20	25	30	35	40
1	45%	40%	15%	62.5	52.2	41.2	28.2	16.4	6.2	0.0	0.0
2	35%	35%	30%	59.7	44.2	33.8	22.4	12.1	3.3	0.0	0.0
3	30%	35%	35%	60.3	41.3	31.1	20.0	10.3	1.9	0.0	0.0
4	20%	35%	45%	60.4	31.9	25.8	17.9	10.2	3.2	0.0	0.0
5	20%	25%	55%	60.3	28.7	21.7	13.3	5.3	0.0	0.0	0.0
6	15%	35%	50%	49.4	14.7	10.5	5.3	0.1	0.0	0.0	0.0
7	15%	25%	60%	49.6	12.6	7.9	2.5	0.0	0.0	0.0	0.0
8	10%	45%	45%	61.2	20.4	14.6	10.2	5.6	0.9	0.0	0.0
9	10%	35%	55%	61.2	18.5	11.6	7.0	2.3	0.0	0.0	0.0
10	10%	25%	65%	59.2	14.4	5.6	0.4	0.0	0.0	0.0	0.0
11	10%	15%	75%	54.9	9.6	0.0	0.0	0.0	0.0	0.0	0.0
12	5%	55%	40%	77.5	31.9	16.7	12.7	8.4	4.0	0.0	0.0
13	5%	45%	50%	78.0	30.0	13.4	9.3	4.9	0.5	0.0	0.0
14	5%	35%	60%	78.0	27.8	9.7	5.5	1.2	0.0	0.0	0.0
15	5%	25%	70%	75.1	23.0	2.9	0.0	0.0	0.0	0.0	0.0
16	5%	15%	80%	64.5	12.3	0.0	0.0	0.0	0.0	0.0	0.0

Table 8: Deviation [in %] of the expected tour length obtained by the single-block layout from the respective shortest expected tour length (class-based demands)

another three classes the deviation exceeds even 75%. The two-block layout (cf. Table 9) provides acceptable deviations for a small number of picks per tour ($k = 5$; 10 and partially for $k = 15$), only. For 24 problem classes the deviations range between 50 and 75%, and for six classes they exceed 75%. In the worst case observed, the expected tour length is almost of double size as the corresponding tour from the best layout (deviation of 93.5% for $k = 40$ and demand distribution of row # 11).

The U-shaped layout (cf. Table 10) provides unacceptable deviations only for some of the problem classes with a small number of picks per tour ($k = 5$). The maximal deviation amounts to 75.2%,

#	demand quan. A	quan. B	quan. C	picks per tour k 5	10	15	20	25	30	35	40
1	45%	40%	15%	0.0	11.3	22.5	29.5	34.3	37.9	44.2	58.6
2	35%	35%	30%	0.0	12.2	25.7	34.3	40.2	44.6	54.5	68.8
3	30%	35%	35%	0.0	11.9	26.5	35.8	42.3	47.1	59.1	73.5
4	20%	35%	45%	0.0	7.3	23.5	34.0	41.5	47.0	56.2	69.6
5	20%	25%	55%	0.0	7.2	24.1	35.3	43.2	51.9	67.3	82.3
6	15%	35%	50%	0.0	3.5	20.4	31.6	39.5	52.9	65.9	78.6
7	15%	25%	60%	0.0	3.1	20.7	32.5	45.0	59.8	74.0	87.7
8	10%	45%	45%	0.0	0.0	13.9	24.5	31.9	37.4	46.7	57.0
9	10%	35%	55%	0.0	0.0	14.0	25.2	33.2	42.5	54.2	65.4
10	10%	25%	65%	0.0	0.0	14.0	25.8	40.8	55.2	68.8	81.8
11	10%	15%	75%	0.0	0.0	13.6	32.6	49.5	65.1	79.6	93.5
12	5%	55%	40%	0.0	0.0	5.5	15.2	22.0	26.9	31.1	39.7
13	5%	45%	50%	0.0	0.0	5.2	15.6	22.9	28.3	37.4	46.9
14	5%	35%	60%	0.0	0.0	4.9	15.9	23.7	33.6	44.2	54.4
15	5%	25%	70%	0.0	0.0	4.1	17.6	32.1	45.3	57.6	69.3
16	5%	15%	80%	0.0	0.0	11.5	30.0	46.0	60.3	73.2	85.2

Table 9: Deviation [in %] of the expected tour length obtained by the two-block layout from the respective shortest expected tour length (class-based demands)

and only for six problem classes ranges the deviation between 50 and 75.2%. Apart from these problem classes (and probably of the classes related to the demand distribution of row # 16), the U-shaped layout – in combination with the Walking-the-U strategy – represents a layout type which provides acceptable tour lengths, even if not the best of the three layout types has been chosen.

	demand			**picks per tour** k							
#	quan. A	quan. B	quan. C	5	10	15	20	25	30	35	40
1	45%	40%	15%	12.0	0.0	0.0	0.0	0.0	0.0	2.5	11.0
2	35%	35%	30%	15.3	0.0	0.0	0.0	0.0	0.0	4.4	12.0
3	30%	35%	35%	17.6	0.0	0.0	0.0	0.0	0.0	5.5	12.8
4	20%	35%	45%	26.8	0.0	0.0	0.0	0.0	0.0	3.2	9.6
5	20%	25%	55%	28.2	0.0	0.0	0.0	0.0	1.9	8.8	15.7
6	15%	35%	50%	34.4	0.0	0.0	0.0	0.0	5.1	10.6	16.1
7	10%	45%	45%	36.2	0.0	0.0	0.0	2.9	8.5	14.2	20.1
8	15%	25%	60%	43.1	2.2	0.0	0.0	0.0	0.0	3.6	8.2
9	10%	35%	55%	45.6	2.9	0.0	0.0	0.0	2.4	7.2	12.2
10	10%	25%	65%	48.3	3.7	0.0	0.0	4.8	10.3	15.8	21.4
11	10%	15%	75%	51.3	4.8	0.0	5.3	10.8	16.4	22.1	27.8
12	5%	55%	40%	56.9	10.6	0.0	0.0	0.0	0.0	0.2	4.5
13	5%	45%	50%	60.5	11.9	0.0	0.0	0.0	0.0	3.8	8.3
14	5%	35%	60%	64.5	13.3	0.0	0.0	0.0	3.1	7.6	12.2
15	5%	25%	70%	69.0	15.1	0.0	1.7	6.6	11.6	16.7	21.8
16	5%	15%	80%	75.2	18.2	9.3	14.2	19.1	24.1	29.0	33.8

Table 10: Deviation [in %] of the expected tour length obtained by the U-shaped layout from the respective shortest expected tour length (class-based demands)

7 Summary and Conclusions

The implementation of an appropriate layout and a corresponding routing scheme is crucial for the efficient operation of an order picking warehouse. Wrong decisions cannot be changed immediately and will therefore have a long-term negative effect on profitability and customer service.

In this article, we compared three different layout types and corresponding routing strategies for order picking warehouses in which slow-moving items are to be picked manually. We presented and verified estimations for the tour lengths for the single-block layout with S-Shape routing, for the two-block layout with Return-with-Replication routing, and for a new layout type, i.e. the U-shaped layout with Walking-the-U routing.

By means of a detailed analysis of the presented estimations it was shown which layout type (in combination with the respective routing scheme) provides the shortest expected tour lengths for different numbers of picks per tour and different assumptions concerning the demand distribution. Generally speaking, the two-block layout is superior for a small and the single-block layout is superior for a large number of picks. The U-shaped layout has proven to be the best layout type for a wide range of medium-size numbers of picks and demand distributions. In comparison to the single- and two-block layouts, the total distance to be travelled by the order picker(s) could be reduced significantly.

Furthermore, the U-shaped layout has proven to be rather insensitive against changes concerning the parameters of the order picking system (number of picks per tour; demand distribution), i.e. the deviations caused by the fact that the U-shaped layout is implemented in situations for which it is not the best one, does not necessarily result in a dramatic increase of the expected tour length.

Finally, as a further advantage, the U-shaped layout requires significantly less space than the single-block layout does. Thus, for manual order picking systems for slow-moving items, the U-shaped layout must be considered as an attractive layout alternative to the more traditional single- and two-block layouts.

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