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A Mathematical Programming Formulation for the Single-Picker Routing Problem in a Multi-Block Layout

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Abstract

The Single-Picker Routing Problem (SPRP) arises in warehouses when items have to be retrieved from their storage locations in order to satisfy a given demand. It deals with the determination of the sequence according to which the requested items have to be picked in the picking area of the warehouse and the identification of the corresponding paths to be travelled by human operators (order pickers). The picking area typically possesses a block layout, i.e. the items are located in parallel picking aisles, and the order pickers can only change over to another picking aisle at certain positions by means of so-called cross aisles. Using this special structure, Scholz et al. (2016) developed a model formulation whose size is independent of the number of locations to be visited. They presented the model for a single-block layout and briefly described how it can be extended to the case of multiple blocks. However, by extending this formulation, the number of variables and constraints is multiplied by the number of blocks and, therefore, the model is not suitable for solving the SPRP in warehouses composed of several blocks. In this paper, the extension to multiple blocks is considered and it is pointed out how to drastically reduce the size of the formulation. Depending on the storage locations of the requested items, the number of variables can be decreased by up to 96%.

Keywords: Traveling Salesman, Order Picking, Picker Routing

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1 Introduction

Warehousing is a critical part of a company's supply chain. Warehouses are necessary to coordinate product flows, to buffer products for a certain time period and to provide customers with requested items (Roodbergen & de Koster, 2001). Every day, a warehouse receives a high amount of items in large lot-sizes which have to be stored and redistributed in small volumes based on thousands of daily customer orders. Bad warehouse performances can cause severe impacts on a company's competitiveness through low levels of customer satisfaction and high costs (Wäscher, 2004). Among the different operations executed in a warehouse, order picking is the most critical one as it is the core of the product flow. Studies have shown that up to 55% of the costs in a warehouse can be allocated to order picking (Tompkins et al., 2010), which deals with the retrieval of requested items from their storage locations in the warehouse (Petersen & Schmenner, 1999). The large proportion of the order picking costs can be attributed to the fact that in many warehouses, human operators (order pickers) are assigned to execute the picking process. This process is mainly composed of traveling through the warehouse, searching for the respective items and picking them from their storage locations, whereof traveling consumes approximately 50% of the total working time of a picker (Tompkins et al., 2010).

In order to reduce the travel time, different procedures can be applied which are improving the allocation of the articles in the warehouse (storage assignment), grouping customer orders into picking orders (order batching) and determining a sequence, in which the order picker can retrieve the items by covering only a short distance (picker routing).

The last mentioned procedure is part of the so-called Single-Picker Routing Problem (SPRP) which deals with finding a tour of minimum length including all storage locations of requested items (Scholz et al., 2016). The SPRP is characterized by the special arrangement of the storage locations in the warehouse which typically follows a so-called block layout (Roodbergen, 2001). Ratliff & Rosenthal (1983) and Roodbergen & de Koster (2001) developed exact algorithms for the SPRP in warehouses composed of one and two blocks, respectively. However, no efficient algorithm is available for the SPRP in warehouses with more than two blocks (Roodbergen, 2001).

Scholz et al. (2016), therefore, designed a model formulation for the SPRP which can be applied to a block layout with an arbitrary number of blocks. They presented the model for a single-block layout and then demonstrated how to extend it to multiple blocks. However, the extension of this formulation leads to a model whose size is multiplied by the number of blocks which may result in increasing computing times and a restricted applicability of the model to SPRPs in warehouses with multiple blocks.

Especially when applied to problems in which only few items have to be picked in a relatively large warehouse, the model formulation of Scholz et al. (2016) leads to quite unsatisfactory results. In order to improve the performance of this approach and to make it applicable to layouts with a larger number of blocks, several possibilities to reduce the size of the mathematical model are considered in this paper. By applying the proposed reduction, it is shown that the size of the model formulation (in terms of number of variables) can be decreased by up to 96%.

The remainder of this paper is organized as follows: The SPRP is introduced in the next section. As it is the basis for the mathematical model, Section 3 contains a brief review of the graph construction according to Scholz et al. (2016). Different circumstances are then considered under which the size of the graph can be reduced significantly (Section 4). Section 5 comprises the core elements of the mathematical model for the SPRP which is designed based on this graph. The paper concludes with a summary and an outlook on further research given in Section 6. The complete model formulation with its several hundred types of constraints is included in the appendix.

2 The Single-Picker Routing Problem

The SPRP represents a special case of the well-known Traveling Salesman Problem (TSP). It consists of finding a tour through the warehouse which starts and ends at the depot and includes the locations of all requested items (pick locations). The tour is performed by an order picker who walks or drives through the warehouse using a picking device and collects the requested items which are specified by customer orders.

The special characteristic of the SPRP can be found in the arrangement of the storage locations in the warehouse which typically follows a block layout. According to this layout, two different types of aisles have to be distinguished, namely picking and cross aisles. Picking aisles run parallel to each other and have to be entered by the order picker in order to retrieve items. Items are stored in racks arranged on both sides of these picking aisles. Cross aisles do not contain any storage locations, but they enable the order picker to enter and exit a picking aisle or to switch between picking aisles. Furthermore, they divide the warehouse into blocks, whereby a block is defined as a part of the warehouse located between two adjacent cross aisles. Correspondingly, a part of a picking aisle between two adjacent cross aisles is denoted as a subaisle. Thus, a warehouse with $q + 1$ cross aisles can be divided into q blocks and includes $q \cdot m$ subaisles, whereby m denotes the number of picking aisles. The corresponding layout is called a q -block layout.

In Fig. 1, a two-block layout with 5 picking aisles is depicted. The rectangles represent the storage locations, while the black rectangles are the pick locations. In this example, the depot is situated in front of the leftmost picking aisle and the blocks are enumerated in ascending order, whereby block 1 is the block nearest to the depot. As mentioned before, a warehouse following a two-block layout contains three cross aisles, namely the front, the rear and a middle cross aisle. The front (rear) cross aisle represents the cross aisle which is nearest (farthest) to the depot. A middle cross aisle separates two blocks from each other and has to be used to change over from one block to another.

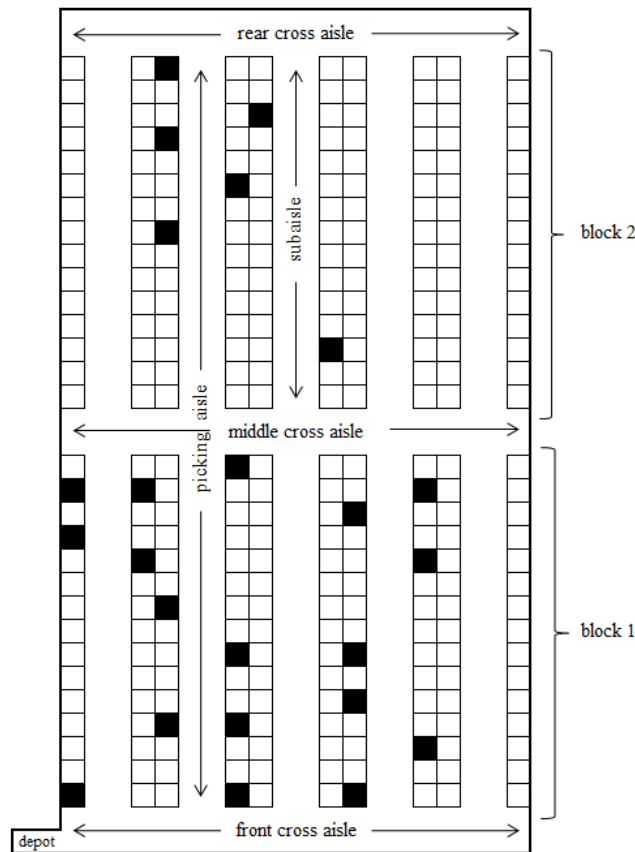


Fig. 1: Two-block layout

Traveling consumes a major part of an order picker's working time, while other components (such as setup times at the depot or picking times at the racks) can be considered to be constant (Caron et al., 2000), as they are independent of the sequence in which the items are picked. Therefore, minimizing the total travel time is a common objective when dealing with the SPRP. Assuming the travel velocity to be constant, the travel time is a linearly increasing function of the travel distance (Jarvis & McDowell, 1991), which means that minimizing the travel time is equivalent to finding a tour of minimum length.

Thus, the SPRP can be defined as follows (Scholz et al., 2016): Given a set of items to be picked from known storage locations, in which sequence should the locations be visited such that the total length of the corresponding tour is minimized?

3 Graph for the SPRP defined by Scholz et al. (2016)

Scholz et al. (2016) developed an approach to deal with the SPRP in a general block layout with an arbitrary number of blocks. This approach is based on a mathematical programming formulation which is obtained by applying a TSP formulation of Gavish & Graves (1978) to a problem-specific graph representing the special structure of the underlying layout.

The authors first used an observation of Burkard et al. (1998) who demonstrated how the SPRP can be formulated as a Steiner TSP. A Steiner TSP is a variant of the TSP in which the set of vertices V can be divided into the two sets P and $V \setminus P$. Thereby, the set P includes the vertices which have to be visited. Vertices contained in the set $V \setminus P$ are Steiner points and are allowed to be skipped. In contrast to the TSP, all vertices are allowed to be visited more than once in a Steiner TSP.

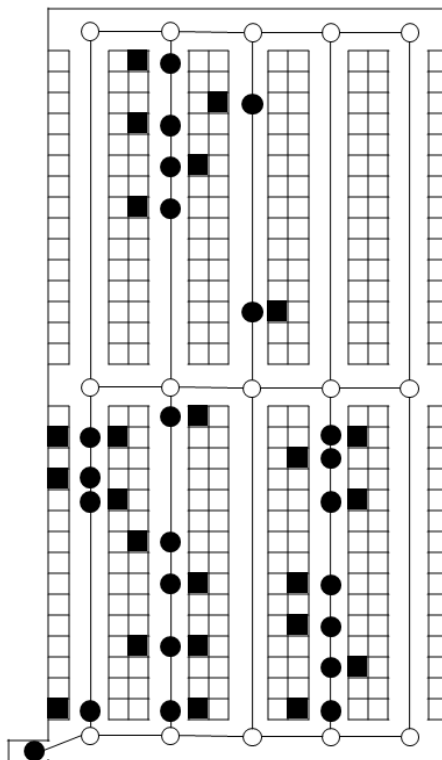


Fig. 2: Illustration of a Steiner TSP

A representation of the SPRP as a Steiner TSP is given in Fig. 2. The black vertices are the vertices which have to be visited, i.e. the vertices contained in the set P . These are the pick locations and the location of the depot. The white vertices represent the Steiner points which do not have to be used. As for the SPRP, these vertices are the intersections between the picking aisles and the cross aisles. Some of these intersections have to be used in order to enter or leave a subaisle. However, since intersections are allowed to be used more than once and, furthermore, not all subaisles necessarily contain requested items, some intersections may not be included in the tour.

Due to the block layout and the resulting missing connections between requested items located in different subaisles, the representation of the Steiner TSP leads to a quite sparse graph. Scholz et al. (2016) additionally considered the movements of an order picker within a subaisle which can be executed in an optimal tour. For constructing an optimal tour, only six possibilities (see Fig. 3) have to be taken into account for collecting items within a subaisle (Ratliff & Rosenthal, 1983).

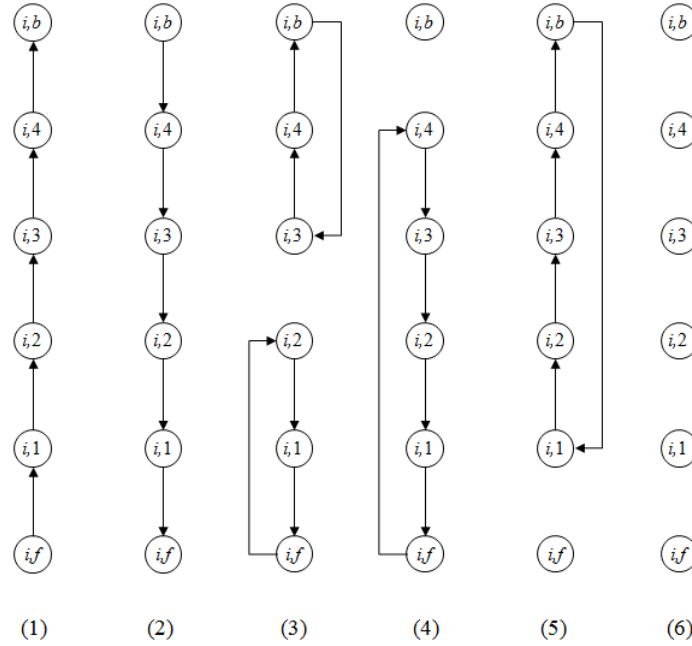


Fig. 3: Movements within a subaisle to be considered for constructing an optimal tour

The order picker can traverse the whole subaisle by entering it from an adjacent cross aisle, visiting the pick locations in that subaisle and leaving it via the other adjacent cross aisle (see Fig. 3 (1) and (2)). Another possibility is to enter and leave a subaisle twice using both adjacent cross aisles. In an optimal tour, this can only be done in such a way that the non-traversed part of the subaisle corresponds to the largest gap which is defined as the largest distance between two adjacent pick locations or a pick location and the adjacent cross aisle. In this case, the order picker enters the subaisle from a cross aisle and returns when he reaches the largest gap. Later, the same procedure is applied using the other cross aisle (see Fig. 3 (3)). The last possibility to collect the requested items in a subaisle is to use the same cross aisle for entering and leaving the subaisle. In this case, the picker enters the subaisle, retrieves all requested items and returns at the pick location which corresponds to the largest distance to the cross aisle from which the subaisle has been entered (see Fig. 3 (4) and (5)). If no requested items are located in a subaisle, the subaisle does not have to be entered at all (see Fig. 3 (6)).

Thus, instead of considering all pick locations and Steiner points, only six vertices are necessary to represent a subaisle (Scholz et al., 2016). These vertices correspond to (see Fig. 3):

- (a) the intersection between subaisle i and an adjacent cross aisle (vertices $[i, f]$ and $[i, b]$),
- (b) the two pick locations defining the largest gap (vertices $[i, 2]$ and $[i, 3]$) and
- (c) the pick locations nearest to an adjacent cross aisle (vertices $[i, 1]$ and $[i, 4]$).

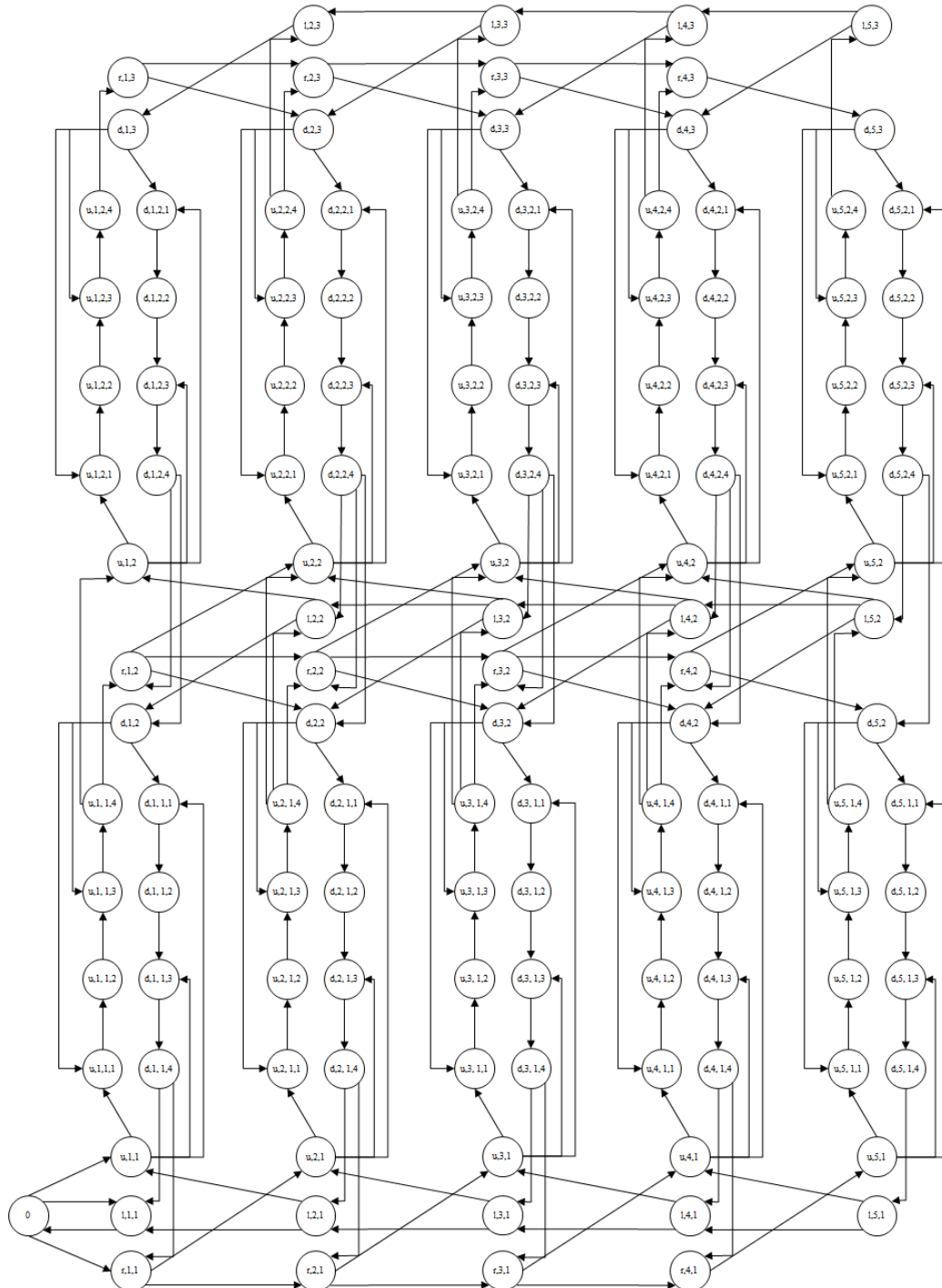


Fig. 4: Graph for a SPRP with two blocks and five picking aisles

Based on this observation, Scholz et al. (2016) were able to construct a graph for the SPRP whose size is independent of the number of pick locations and only dependent on the number of picking aisles. The graph is obtained by introducing the vertices and arcs depicted in Fig. 3 for each subaisle.

Furthermore, arcs representing movements in cross aisles are added. In a second step, in order to apply a TSP formulation to this graph, they modified the graph to ensure that each vertex is visited at most once by copying each vertex several times. The number of copies is determined based on how often a vertex can be visited in an optimal tour. This leads to one copy for each vertex representing a pick location, two copies for vertices in the front or the rear cross aisle and three copies for vertices corresponding to intersections between a subaisle and a middle cross aisle. The resulting graph is depicted in Fig. 4.

The vertices are denoted in the following way: The first entry indicates the direction in which the tour can be proceeded after visiting the vertex. Thereby, "r" and "l" symbolize that the next step will be a movement to the right and to the left, respectively. Movements towards the rear and the front cross aisle are indicated by "u" ("up") and "d" ("down"). The second component is the number of the picking aisle to which the vertex corresponds. The leftmost picking aisle is denoted as picking aisle 1 while the rightmost aisle is picking aisle m . For vertices corresponding to pick locations, the third component characterizes the number of the block and, furthermore, these vertices have a fourth component indicating the position of the vertex in the respective subaisle. The third component of vertices representing an intersection between a subaisle and a cross aisle indicates the number of the cross aisle. The cross aisles are enumerated from 1 to $p + 1$, where p is the number of blocks and cross aisle 1 is the cross aisle nearest to the depot. Finally, vertex "0" represents the location of the depot.

4 Considerations to reduce the size of the graph

The size of the resulting graph is only dependent on the number of picking aisles and not on the number of pick locations, which is an advantage if a lot of items have to be collected. Scholz et al. (2016) demonstrated that the resulting model formulation clearly outperforms general TSP or Steiner TSP formulations if the ratio n/m (whereby n is the number of pick locations and m denotes the number of picking aisles) is not too small. They tested their model formulation on problems with a single-block layout and pointed out that the advantage of the formulation diminishes when only a few items have to be picked per picking aisle. This can be explained by the fact that each picking aisle is represented by a constant number of vertices regardless of the number of pick locations in this aisle which may not be a huge problem for the case of a single-block layout. However, when considering a multi-block layout, a picking aisle consists of several subaisles each of which requiring a constant number of vertices to be represented. When dealing with a layout including p blocks, the size of the graph (in terms of the number of vertices and arcs) is approximately multiplied by p . Since each arc results in a binary and a

real valued variable in the model formulation of Scholz et al. (2016), the size of this formulation will strongly increase if a larger number of blocks is considered which may result in a limited applicability of the model.

Therefore, we now focus on some possibilities to drastically reduce the size of the graph. We will first show how the graph can be reduced to a "pyramid structure" by removing all vertices from the graph corresponding to certain subaisles. Second, we will deal with some special cases in which the pick locations are distributed over a subaisle in a certain manner and demonstrate how the pick locations can be represented by using less vertices and arcs.

4.1 The pyramid structure

In large warehouses containing a high number of racks (and subaisles), lots of different articles can be stored. However, if articles are randomly assigned to storage locations, order picking tours may become quite long. In order to reduce the distance to be covered for retrieving requested items, other procedures for the assignment of articles to storage locations are applied. The class-based storage assignment is one example for those procedures in which articles with a high expected demand are stored near the depot (Petersen & Schmenner, 1999). Since articles assigned to storage locations far away from the depot have a very low demand, only subaisles close to the depot have to be visited in most tours.

When constructing an optimal order picking tour, generally all subaisles have to be considered in order to not exclude tours which might be optimal ones. However, if no subaisle located far away from the depot includes requested items, the order picker will never visit this part of the warehouse and, therefore, it may be possible to exclude these subaisles but still guaranteeing to find an optimal tour. In the following, it is shown which criteria have to be fulfilled to ensure that no optimal tour is excluded when a certain subaisle is removed from the graph.

Let $B = \{1, \dots, p\}$ be the set of blocks and $\overline{\mathcal{M}}_q$ the rightmost subaisle of block $q \in B$ containing at least one requested item. Furthermore, let \overline{m}_q denote the rightmost subaisle of block $q \in B$ which has to be included in the graph in order to construct an optimal order picking tour. Obviously, it must hold $\overline{m}_q \geq \overline{\mathcal{M}}_q$ for each block $q \in B$ since each subaisle containing a requested item has to be visited.

Another reason for a subaisle to be visited is to change over to another cross aisle in order to go to an adjacent block. Consider a SPRP with $\overline{\mathcal{M}}_1 = \overline{\mathcal{M}}_3 = \tilde{m}$ and $\overline{\mathcal{M}}_2 = \tilde{m} - 1$ with $\tilde{m} \in \{2, \dots, m\}$. If the order picker visits subaisle \tilde{m} of block 1 and then has to go to subaisle \tilde{m} of block 3, the shortest path would be to traverse subaisle \tilde{m} of block 2. Removing this subaisle from the graph would cause a detour,

although both, this subaisle and all other subaisles of this block located on the right of this subaisle, do not contain requested items. Thus, a subaisle of a block $q \in B$ must not be removed if both, the adjacent lower and upper block, include subaisles which have to be considered and are located further on the right, i.e. if it holds $\overline{\mathcal{M}}_{q-1} > \overline{\mathcal{M}}_q$ and $\overline{\mathcal{M}}_{q+1} > \overline{\mathcal{M}}_q$.

By using this observation, the size of the graph can be reduced introducing a pyramid structure, which means that \overline{m}_q is determined by solving the following mathematical program for each block $q \in B$ (whereby $\overline{m}_{p+1} := 0$).

$$\min \overline{m}_q \quad (1)$$

$$\overline{m}_q \geq \overline{\mathcal{M}}_q \quad (2)$$

$$\overline{m}_q \geq \overline{m}_{q+1} \quad (3)$$

The objective function minimizes the index of the rightmost subaisle to be considered in block q . Constraint (2) guarantees that no subaisle of block q containing at least one requested item is removed from the graph. The pyramid structure is constructed by constraint (3) which ensures that the index of the rightmost subaisle to be considered is not larger for block $q+1$ than for block q . This constraint guarantees that no optimal solution is excluded by removing subaisles of block q from the graph.

Since $\overline{m}_{p+1} = 0$, the optimal solution for block p is $\overline{m}_p = \overline{\mathcal{M}}_p$. Then, \overline{m}_q can be determined successively for the other blocks $q \in B \setminus \{p\}$:

$$\overline{m}_q = \max \{ \overline{\mathcal{M}}_q; \overline{m}_{q+1} \}. \quad (4)$$

An analogue procedure can be applied to subaisles located in the first subaisles of a block. Let $\underline{\mathcal{M}}_q$ be the leftmost subaisle of block $q \in B$ containing a pick location and \underline{m}_q the leftmost subaisle of block $q \in B$ which has to be considered for constructing an optimal tour. With the same line of argumentation as above, we now determine \underline{m}_q for each block $q \in B$ which results in $\underline{m}_p = \underline{\mathcal{M}}_p$ and for each block $q \in B \setminus \{1, p\}$ we obtain

$$\underline{m}_q = \min \{ \underline{\mathcal{M}}_q; \underline{m}_{q+1} \}. \quad (5)$$

The only difference between these two procedures can be seen in the first block. As for the determination of \overline{m}_1 , formular (4) is applied, formular (5) cannot be used to compute \underline{m}_1 . Since we assume that the depot is located in front of the leftmost picking aisle, subaisle 1 of the first block has to be considered ensuring a connection to the depot. Thus, even if the first picking aisle does not contain any pick locations, \overline{m}_1 has to be set to 1.

After determining \bar{m}_q (\underline{m}_q) for each block $q \in B$, all subaisles of each block q located further to the right (left) than subaisle \bar{m}_q (\underline{m}_q) are removed from the graph. When removing a subaisle i ($1 < i < m$) of block $q \in B \setminus \{1, p\}$ from the graph, all arcs corresponding to movements in this subaisle are deleted resulting in a reduction of 18 arcs. Since this subaisle will not be entered in an optimal tour, the vertices $[u, i, q]$ and $[d, i, q + 1]$ are not needed anymore leading to a reduction of another 6 arcs. Furthermore, due to the pyramid structure, the part of cross aisle $q + 1$ situated directly above subaisle i does not have to be used in order to construct an optimal tour. The vertices $[r, i, q + 1]$ and $[l, i, q + 1]$ and the remaining 6 arcs adjacent to one of these vertices can thus be removed from the graph as well.

Summing up, by applying this procedure, we reduce the size of the original graph by approximately $\sum_{q \in B} 30 \cdot [(\underline{m}_q - 1) + (m - \bar{m}_q)]$ arcs while ensuring that no optimal order picking tour is excluded.

4.2 Special cases of item distribution

The model formulation of Scholz et al. (2016) works very well for SPRPs with a large number of requested items compared to the number of subaisles since eight vertices are always used to represent the pick locations in a subaisle regardless of the real number of pick locations in that aisle. If less vertices are required for describing the pick locations, Scholz et al. (2016) introduce some dummy vertices. Though this procedure ensures that the size of the graph is completely independent of the number of pick locations, many vertices and arcs may be required for representing only a few pick locations. When dealing with a single-block layout, this fact has not to be considered since the number of pick locations per subaisle is usually sufficiently large in this situation. However, in the case of multiple blocks, the number of subaisles is multiplied by the number of blocks and the pick locations are distributed over a large number of subaisles. Thus, lots of dummy vertices and arcs will be introduced if the approach of Scholz et al. (2016) is applied increasing the size of the resulting model formulation.

In the following, we consider different cases in which the number of vertices and arcs required for representing the pick locations in a certain subaisle can be reduced. For the sake of simplicity of exposition, we will focus on a subaisle i of block $q \in B \setminus \{1, p\}$ with $\underline{m}_{q+1} < i < \bar{m}_{q+1}$. In the standard case, 8 vertices are used for representing the pick locations and the graph includes 18 arcs incident to at least one of these vertices (see Fig. 4). Since two variables are introduced in the formulation of Scholz et al. (2016) for each arc, we focus on the reduction of the number of arcs contained in the graph.

The largest reduction can be observed when a subaisles does not contain any pick locations. In this case, arcs are only required in order to ensure that this subaisle can be used to switch over to an adjacent cross

aisle. On the left hand side of Fig. 5, the subaisle including the storage locations is depicted while the corresponding part of the graph is shown on the right hand side. For entering the cross aisle, either the vertex $[u, q, i]$ or $[d, q + 1, i]$ has to be visited. The order picker then can proceed his tour by going to the left, to the right or by entering a subaisle of an adjacent block. In order to represent these possible moves, 6 arcs are needed which gives a reduction of 12 arcs per subaisle.

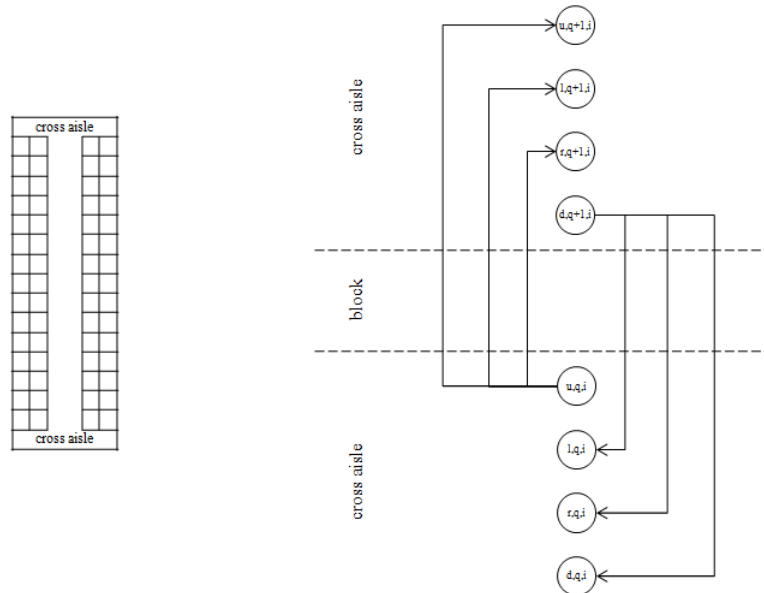


Fig. 5: Special Case 1 - Subaisle containing no pick locations

Another reduction of the number of arcs can be obtained by considering the location of the largest gap in a subaisle. If the largest gap lies between cross aisle q and the adjacent pick location, then the vertex $[u, q, i]$ and the vertex pair $([u, q, i, 2], [d, q, i, 3])$ will represent the same location (see Fig. 4). In this case, the pick locations in this subaisle can be represented by only using two vertices (see Fig. 6).

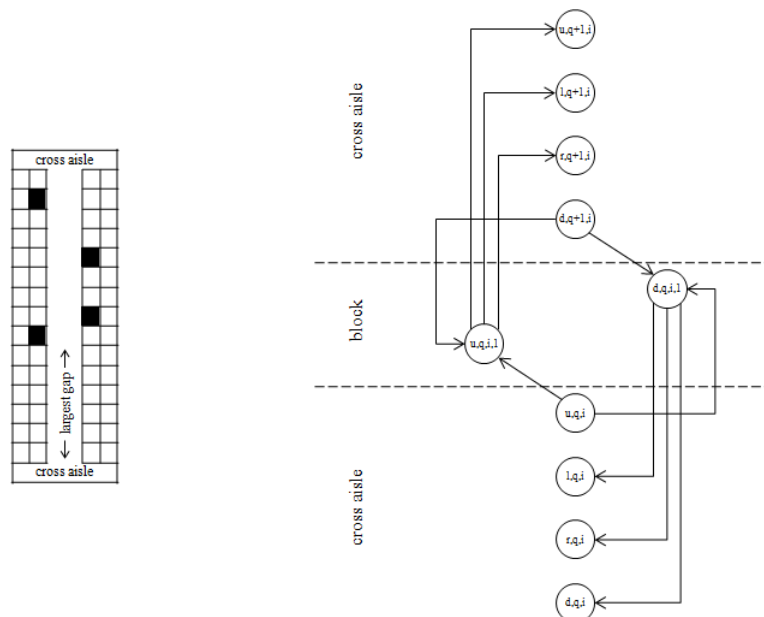


Fig. 6: Special Case 2 - Subaisle with largest gap between a pick location and the adjacent cross aisle

Vertex $[u, q, i, 1]$ represents the pick location nearest to cross aisle q which also defines "the end" of the largest gap. The location of the requested item farthest from cross aisle q is given by vertex $[d, q, i, 1]$. In this case, we have single vertices and no vertex pair which means that the distance to vertex $[u, q, i]$ may be different for both vertices. This is caused by the fact that applying a move regarding the largest gap strategy (see Fig. 3 (3)) is the same as performing a return strategy from cross aisle $q + 1$ (see Fig. 3 (4)) here. For representing such a subaisle, 10 instead of 18 arcs are required. The same line of argumentation holds if the largest gap lies between cross aisle $q + 1$ and the adjacent pick location.

A very simple possibility to reduce the size of the graph arises when only two pick locations are contained in a subaisle while the largest gap lies between these two locations. Then, a vertex pair is introduced for each pick location and the arcs are chosen in such a way that all strategies included in Fig. 3 can be performed. 14 arcs are needed to represent a subaisle in this case (see Fig. 7).

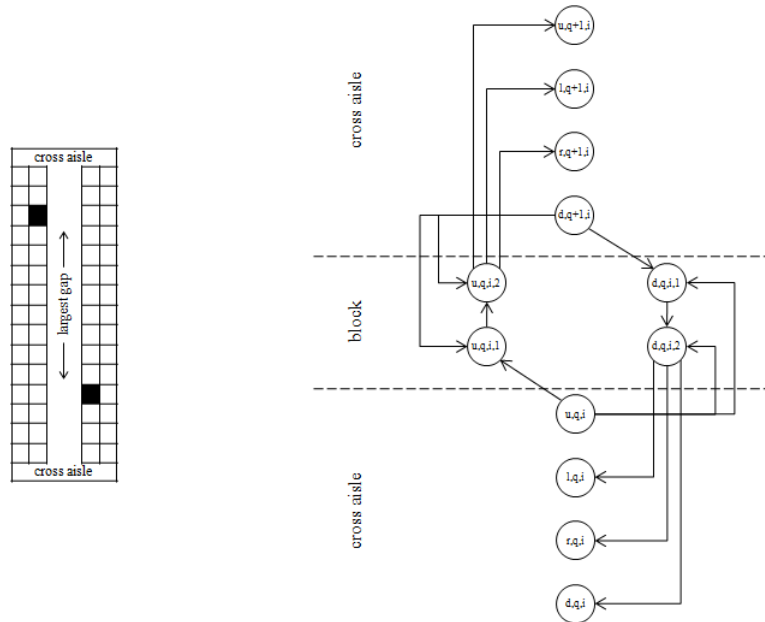


Fig. 7: Special Case 3 - Subaisle with two pick locations and largest gap between them

A slight reduction of the size of the graph can be achieved when the pick location adjacent to cross aisle q defines "the beginning" of the largest gap in this subaisle. Considering the standard case depicted in Fig. 4, the vertex pairs $([u, q, i, 1], [d, q, i, 4])$ and $([u, q, i, 2], [d, q, i, 3])$ would define the same location and, therefore, one pair can be neglected. The resulting representation of a subaisle can be seen in Fig. 8. By removing a vertex pair, two arcs can be removed from the graph as well, resulting in 16 instead of 18 arcs required for representing such a subaisle.

Analogously, two arcs can be removed when the pick location adjacent to cross aisle $q + 1$ defines "the end" of the largest gap. This case is denoted by special case 5. The only difference to special case 4 is that the arc between the vertices $[d, q + 1, i]$ and $[u, q, i, 2]$ is replaced by an arc between $[u, q, i]$ and

$[d, q, i, 2]$ which is caused by the changing of the position of the largest gap.

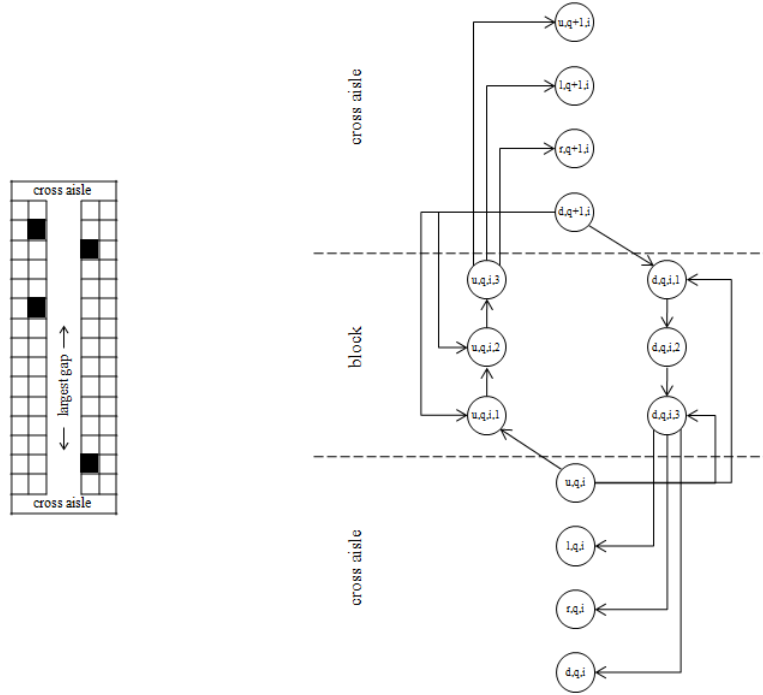


Fig. 8: Special Case 4 - Subaisle with largest gap between the two pick locations nearest to cross aisle q

4.3 A comparison between the initial and the reduced graph

The initial graph for the SPRP depicted in Fig. 1, which is constructed according to the approach of Scholz et al. (2016), includes 217 arcs (see Fig. 4), while the size of the graph is neither dependent on the number nor on the location of the requested items.

Applying the considerations above, we can first reduce the size of the graph due to the pyramid structure. In block 2, only the second and the third subaisle contain requested items which results in $\underline{\mathcal{M}}_2 = 2$ and $\overline{\mathcal{M}}_2 = 3$. The fourth subaisle is the rightmost subaisle with pick locations in block 1 and, thus, $\overline{\mathcal{M}}_1 = 4$. Since the depot is located in front of picking aisle 1, $\underline{\mathcal{M}}_1$ is equal to 1. Applying formulas (4) and (5) then leads to $\underline{m}_1 = 1$, $\underline{m}_2 = 2$, $\overline{m}_1 = 4$ and $\overline{m}_2 = 3$ which implies that vertices and arcs corresponding to subaisle 5 of block 1 or to the subaisles 1, 4 and 5 of block 2 can be removed from the graph.

Considering the special cases of item distribution, further vertices and arcs can be identified which are removable. Starting with block 1, the largest gap in the leftmost subaisle is situated between the pick location nearest to the front cross aisle and an adjacent pick location which is special case 4. The analogue case (special case 5) can be observed in subaisle 2. The next subaisle does not contain any requested items and, therefore, can be treated according to special case 1. The rightmost subaisle to be considered in this block follows the standard case in which the pick locations defining the largest

gap and the pick location nearest to the cross aisles represent different locations which implies that no vertices and arcs can be removed here. In block 2, the subaisles follow the special cases 2 (subaisle 2) and 3 (subaisle 3) since the largest gap is located between a pick location and the adjacent cross aisle (subaisle 2) or between two pick locations while only two pick locations exist in this subaisle (subaisle 3). The resulting reduced graph is depicted in Fig. 8 and contains 98 arcs which is a reduction of 55% compared to the original graph.

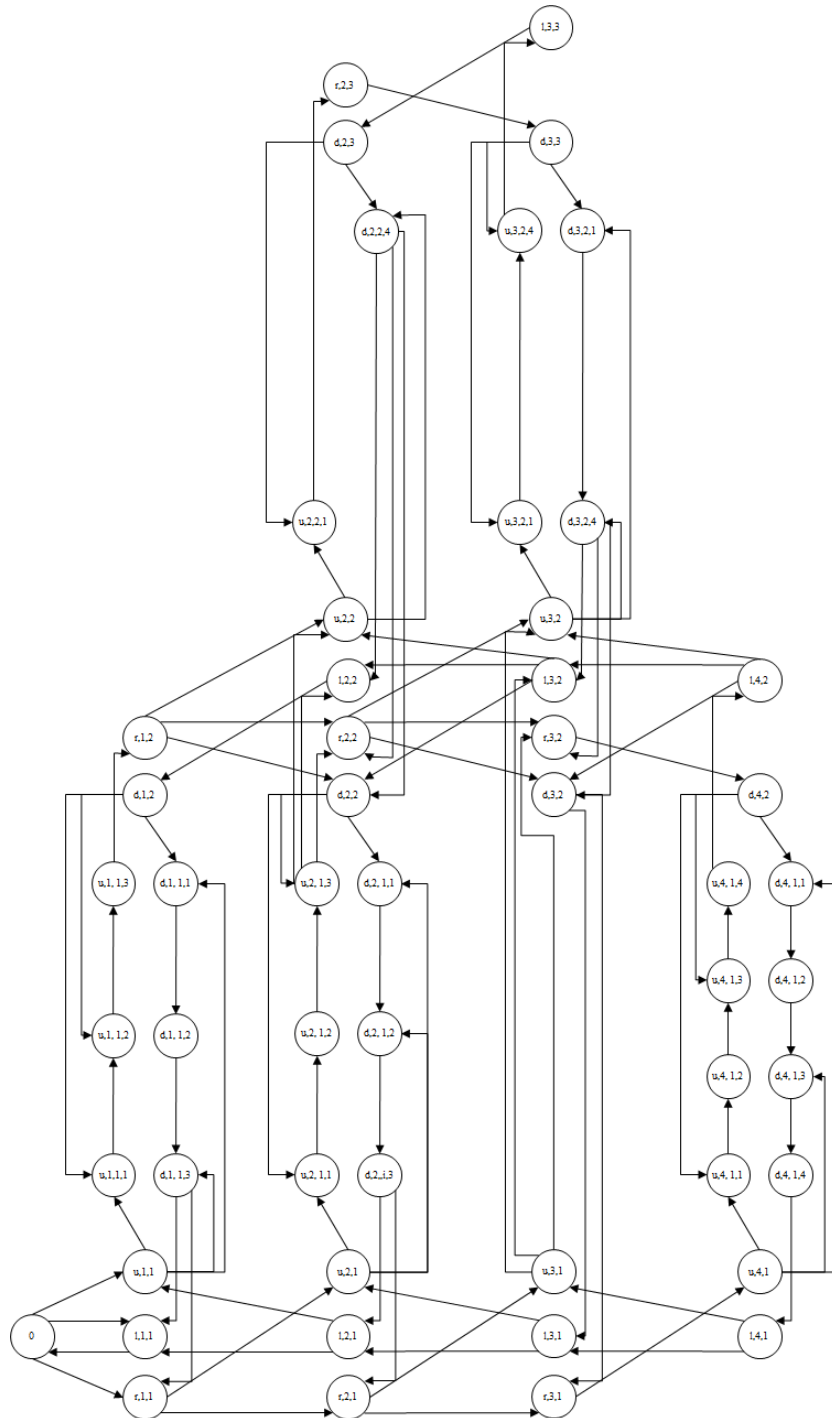


Fig. 9: Reduced graph for a SPRP with two blocks

For a more general investigation of the impact of the reduction, we consider the largest problem class

from the numerical experiments conducted by Scholz et al. (2016). They deal with a single-block layout with up to 30 picking aisles and 90 pick locations. Extending the graph to a multi-block layout, it includes 1418 (two-block layout) or 2126 (three-block layout) arcs. Since the size of the reduced graph is dependent on the location of the requested items, we can only provide worst and best case reductions.

According to the pyramid structure, the worst case arises when both the leftmost and the rightmost subaisle of block p contain at least one pick location. The largest reduction (best case) can be obtained if the leftmost subaisle of block 1 is the only one containing requested items. However, in order to have a fair comparison, we assume that each block includes at least three subaisles to be considered.

The maximum number of arcs required for representing the pick location can be determined by comparing the special cases with respect to their maximum number of arcs per pick location. In Table 1, the number of arcs (#arcs), the minimum number of pick locations (#locations) required to obtain the special case and the maximum number of arcs per pick location are given for the special cases of item distribution and the standard case (special case 0).

Table 1: Maximum number of arcs required for representing pick locations in a subaisle

special case	0	1	2	3	4	5
#arcs	18	6	10	14	16	16
#locations	4	0	1	2	3	3
#arcs/#locations	4.5	-	10	7	5.3	5.3

As it can be seen in Table 1, the maximum number of arcs per pick location is required if a subaisle contains exactly one requested item. In this case, 10 arcs are needed to represent the subaisle. In order to construct the worst case scenario, as many subaisles as possible are generated including only one pick location. Since we consider problem instances with 90 pick locations, the number of pick locations may be larger than the number of subaisles which is equal to 60 (two-block layout) or 90 (three-block layout). Therefore, if a two-block layout is considered, some subaisles have to contain two pick locations resulting in special case 3 which is the second worst case with respect to the maximum number of arcs per pick location. For the worst case scenario, a warehouse with two (three) blocks is then composed of 30 (0) subaisles with two pick locations and 30 (90) subaisles containing one requested item.

For constructing the best case scenario, exactly two subaisles with pick locations are generated while all other subaisles are empty. Two subaisles are required since it is assumed that three subaisles per block have to be considered. This can be obtained by considering the three leftmost subaisles in the last block from which only subaisles 1 and 3 have to contain requested items. The items are distributed in such a way that special case 2 arises in both subaisles.

The number of arcs required for representing a SPRP with two or three blocks according to the approach of Scholz et al. (2016) and by using the reduced graph is depicted in Table 2. For the reduced graph, the number of arcs is shown for the worst and the best case scenario with respect to the distribution of the pick locations as explained above.

Table 2: Number of arcs included in the initial and reduced graph

#blocks	Scholz et al. (2016)	Reduced graph: worst case	Reduced graph: best case
2	1418	1058	58
3	2126	1406	82

Comparing the size of the initial and the reduced graph, it can be observed that even in the worst case scenario the number of arcs is decreased by 25.4% (two-block layout) and 33.9% (three-block layout), respectively. However, an item distribution according to the worst case would cause a very long tour since the order picker would have to visit each subaisle in order to retrieve the requested items. To avoid this and to reduce the length of the tours, items are usually stored in a specific way instead of randomly assigning them to storage locations. Therefore, it is more likely that only some of the subaisles have to be visited which further reduces the size of the graph. When assuming that up to three subaisles have to be considered in each block, the number of arcs in the initial graph constructed according to Scholz et al. (2016) can be reduced by up to approximately 96%. Since the number of variables of the resulting mathematical model is only determined by the number of arcs in the graph, the size of the model can be decreased drastically by formulating the model based on the reduced graph.

5 A model formulation based on the reduced graph

In general, the mathematical model formulation can be applied to any block layout. However, due to the pyramid structure and the special cases of item distribution introduced in the previous section, the model is composed of several hundred types of constraints. In order to limit a further distinction of cases, we make the following two assumption with respect to the layout and the pick locations:

- The warehouse follows a multi-block layout, i.e. the number of blocks p is not smaller than 2.
- At least three subaisles of block p are contained in the reduced graph.

For each arc of the reduced graph, a binary variable is introduced which takes the value 1 if the arc is contained in the tour and the value 0 otherwise. The denotation of these variables is explained by means of a specific picking aisles i in a two-block layout which is depicted in Fig. 9. (Note that both subaisles

in Fig. 9 are assumed to follow the standard case of item distribution and that some arcs regarding movements in cross aisles are missing.)

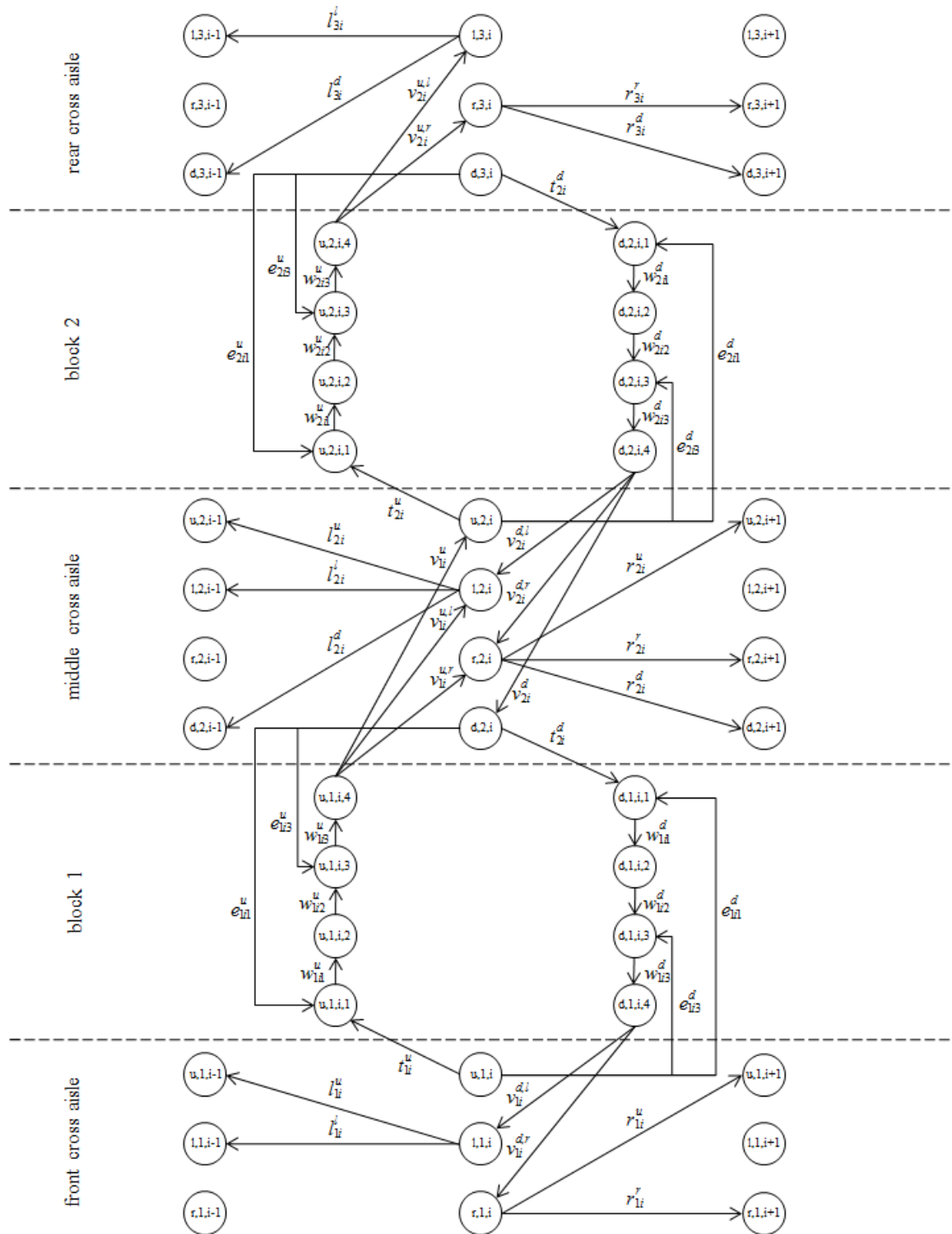


Fig. 10: Denotation of the variables

The variables are denoted as follows: The symbol itself describes the type of the movement, whereby it is distinguished between movements to the right ("r"), to the left ("l"), movements between pick locations within subaisles ("w") and movements to enter ("e") or leave ("v") a subaisle. An exception can be

seen regarding movements to enter a subaisle since those movements with the purpose to traverse the subaisle are denoted by "t". The upper index of the respective symbol specifies the next move, whereby movements upwards ("u"), downwards ("d"), to the right ("r") and to the left ("l") are possible. The two lower indices indicate the corresponding cross aisle or block and the subaisle, respectively. Furthermore, some variables have an additional index which is simply the number of the vertex defining the end of the arc. Following this denotation, for example, the variable e_{2i1}^u represents the arc corresponding to enter subaisle i using cross aisle 2. Thereby, vertex 1 of this subaisle is visited first and afterwards the next movement will go upwards.

Furthermore, real valued variables are introduced for each arc in order to prohibit subtours using single commodity flow constraints as it is done by Scholz et al. (2016).

Besides the objective function (6), which minimizes the total length of the tour, the mathematical programming formulation has to contain constraints ensuring the tour to start from and end at the depot while including all pick locations. These constraints can be divided into the following classes:

- Depot inclusion constraint [(7)]: The depot has to be a part of the tour.
- Pick location inclusion constraints [(8) - (16)]: All pick locations have to be visited.
- Degree constraints [(17) - (228)]: Each vertex visited has to be left afterwards.
- Subtour elimination constraints [(229) - (465)]: The tour has to be connected.
- Variable domain constraints [(466) - (513)]: The variables have to be binary or real valued.

The first two classes of constraints deal with the locations to be included in an order picking tour. In order to guarantee that the depot is a part of the tour, constraint (7) ensures that the depot will be left.

$$y_l^0 + y_r^0 + y_u^0 \geq 1 \quad (7)$$

This is done by forcing the outdegree of vertex "0", which represents the location of the depot, to be larger than or equal to 1. In conjunction with the objective function, this constraint ensures that exactly one of the arcs leaving the depot is included in the tour.

In order to construct tours in which all pick locations are visited, it has to be ensured that the corresponding subaisles are entered and left in such a way that all items can be retrieved. Assuming the standard case of item distribution, all requested items in a subaisle are located between the pick locations nearest to a cross aisle and the corresponding adjacent location defining the beginning of

the largest gap. No pick locations are situated between the two points defining the largest gap. Thus, by using either arc $([u, q, i, 1], [u, q, i, 2])$ or $([d, q, i, 3], [u, q, i, 4])$ and either arc $([u, q, i, 3], [u, q, i, 4])$ or $([d, q, i, 1], [u, q, i, 2])$, all pick locations in a subaisle i of a block q will be visited. This is ensured by introducing constraints (8) and (9) which guarantee that at least one of the two corresponding variables is set to 1, respectively.

$$w_{qi1}^u + w_{qi3}^d \geq 1 \quad \forall q \in B, i \in I_0^q \quad (8)$$

$$w_{qi3}^u + w_{qi1}^d \geq 1 \quad \forall q \in B, i \in I_0^q \quad (9)$$

Similar considerations have to be done for the special cases of item distribution (except for subaisles following special case 1 since those subaisles do not contain any requested items). For a subaisle i of block q in which the items to be picked are distributed according to special case 2, all pick locations are situated between the locations defined by the vertices $[u, q, i, 1]$ and $[d, q, i, 1]$. In this case, the arcs are arranged in such a way that visiting and leaving one of these two vertices corresponds to the application of the traversal or return strategy to this subaisle which means that all requested items are retrieved in this subaisle. Thus, it has to be ensured that either vertex $[u, q, i, 1]$ or vertex $[d, q, i, 1]$ is included in the tour. This is done by constraints (10) which require the sum of their indegrees to be larger than or equal to 1.

$$t_{qi}^u + e_{qi1}^d + t_{qi}^d + e_{qi1}^u \geq 1 \quad \forall q \in B, i \in I_2^q \quad (10)$$

Since subaisles assigned to special case 3 only contain two pick locations, it has to be guaranteed that both pick locations are included in the tour. This is done by ensuring that one vertex of the vertex pair representing the pick location is visited, respectively, which results in constraints (11) and (12).

$$w_{qi1}^u + w_{qi1}^d + e_{qi2}^d \geq 1 \quad \forall q \in B, i \in I_3^q \quad (11)$$

$$w_{qi1}^u + w_{qi1}^d + e_{qi2}^u \geq 1 \quad \forall q \in B, i \in I_3^q \quad (12)$$

Due to the construction of the graph, it is sufficient to ensure that the two locations defining the largest gap are included in the tour if a subaisle i of block q belongs to special case 4. This is equivalent to guarantee that at least one vertex of the vertex pair representing such a point will be visited, resulting in constraints (13) and (14). The same line of argumentation holds for subaisles assigned to special case 5. For those subaisles, constraints (15) and (16) ensure that all pick locations will be visited.

$$w_{qi1}^u + w_{qi2}^d + e_{qi3}^d \geq 1 \quad \forall q \in B, i \in I_4^q \quad (13)$$

$$w_{qi2}^u + w_{qi1}^d \geq 1 \quad \forall q \in B, i \in I_4^q \quad (14)$$

$$w_{qi1}^u + w_{qi2}^d \geq 1 \quad \forall q \in B, i \in I_5^q \quad (15)$$

$$w_{qi2}^u + e_{qi3}^u + w_{qi1}^d \geq 1 \quad \forall q \in B, i \in I_5^q \quad (16)$$

The remaining three classes of constraints are standard TSP constraints required for generating a tour which is connected. The degree constraints ensure that a vertex visited will be left afterwards. In constraints [(17)-(228)] this is done by forcing the indegree of a vertex (right hand side of the constraints) to be equal to its outdegree (left hand side). The subtour elimination constraints [(229)-(465)] are so-called single commodity flow constraints introduced by Gavish & Graves (1978) and adapted by Letchford et al. (2013) and Scholz et al. (2016). The idea is that the picker starts the tour with M units of a single commodity (whereby M denotes the number of vertices in the graph) and delivers one unit to each vertex visited. Following this procedure, the vertices (or rather the arcs used to visit the vertices) are enumerated according to the sequence in which they appear in the tour excluding subtours. The left hand side of the constraints determines the difference of the amount of the commodity delivered to and leaving the vertex. This amount has to be equal to 1 for all vertices included in the tour. Since not all vertices in the underlying graph have to be visited, we calculate the outdegree of a vertex (right hand side of the constraints) in order to make sure that one unit of the commodity is delivered to a vertex if and only if it is a part of the tour.

The model formulation with its more than 500 types of constraints does not seem to be applicable to problem instances arising in practice. However, as it is the case for the formulation of Scholz et al. (2016), the size of the formulation only increases linearly with the number of subaisles. The size is furthermore limited with respect to the number of pick locations since the number of arcs required for representing the movements within a subaisle is limited by 18 (see standard case of item distribution). This makes the model formulation advantageous over general and Steiner TSP formulations if the number of pick locations gets large. The large number of different constraint types is caused by the application of the pyramid structure and the consideration of different cases regarding the distribution of requested items in a subaisle. However, it has been shown that the size of the formulation can be drastically decreased by considering these two aspects (see Section 4). Moreover, when applying the model formulation to a specific problem instance, only a fraction of these constraints may appear in the corresponding mathematical model.

6 Conclusion and Outlook

The Single-Picker Routing Problem (SPRP) deals with the determination of the sequence according to which an order picker has to retrieve requested items from their storage locations in the warehouse. Due to the structure of the warehouse's layout, optimal tours are very restricted with respect to the movements within a subaisle. Based on this observation, Scholz et al. (2016) developed a problem-specific mathematical model for the SPRP which they tested on instances with a single-block layout showing that the application of this model formulation outperforms TSP formulations by far in terms of computing times, especially for a high ratio between the number of pick locations and the number of subaisles.

However, dealing with a multi-block layout, as it is the case in this paper, increases the number of subaisles, resulting in a smaller ratio if the number of pick locations is fixed. Moreover, the size of the model formulation of Scholz et al. (2016) is multiplied by the number of blocks which limits the applicability of the model to layouts with multiple blocks. In this paper, it is shown how the size of this formulation can be decreased significantly by reducing the size of the underlying graph for the SPRP. Both, the introduction of a pyramid structure, by which whole subaisles can be removed from the graph, and the consideration of different cases regarding the locations of requested items in a subaisle, are used to reduce the size of the graph. A comparison between the initial and the reduced graph reveals that the number of arcs (and, therefore, the number of variables in the model formulation) can be decreased by up to 96% for the largest problem class considered by Scholz et al. (2016). Based on the reduced graph, an improved model formulation is given for the SPRP with multiple blocks which includes far less variables than the formulation of Scholz et al. (2016).

The next step will be to conduct numerical experiments in order to investigate the effect of the size reduction. On the one hand, the model formulation should be compared to the formulation of Scholz et al. (2016) and on the other hand, TSP and Steiner TSP formulations should also be taken into account since these formulations outperform the model of Scholz et al. (2016) when the number of subaisles is very large compared to the number of pick locations.

A further interesting topic for future research would be the integration of valid inequalities in order to extend the applicability of the model and reduce the computing times required for solving it. For example, in an optimal solution, only one routing strategy will be applied to a subaisle which could be a general concept for developing valid inequalities. The investigation of symmetry breaking constraints would also be a promising point since there always exist at least two optimal tours which makes proving the optimality of a solution quite difficult.

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Appendix: Model formulation for the SPRP

Sets:

- $B = \{1, \dots, p\}$: set of blocks
- $F = \{1, \dots, p+1\}$: set of cross aisles
- $I^q = \{\underline{m}_q, \overline{m}_q\}$: set of subaisles to be considered in block q , with $\underline{m}_1 = 1$ and $\underline{m}_q < \overline{m}_q - 1 \forall q \in B$ (see Section 4.1 for the determination of \underline{m}_q and \overline{m}_q)
- $I_0^q \subseteq I^q$: subset of I^q with subaisles in block q that contain requested items distributed according to the standard case of item distribution ($I_0^q = I^q \setminus \{I_1^q \cup I_2^q \cup I_3^q \cup I_4^q \cup I_5^q\}$)
- $I_1^q \subseteq I^q$: subset of I^q with subaisles in block q that contain no requested items (special case 1)
- $I_2^q \subseteq I^q$: subset of I^q with subaisles in block q that contain the largest gap between the first or last requested item and the adjacent cross aisle (special case 2)
- $I_3^q \subseteq I^q$: subset of I^q with subaisles in block q that contain exactly two requested items with the largest gap located between these two items (special case 3)
- $I_4^q \subseteq I^q$: subset of I^q with subaisles in block q that contain the largest gap between the first requested item and the other requested items (special case 4)
- $I_5^q \subseteq I^q$: subset of I^q with subaisles in block q that contain the largest gap between the last requested item and the other requested items (special case 5)

Binary variables indicating the arcs included in the tour:

r_{qi}^r : binary variable, $\forall (q, i) \in F \times (I^q \setminus \{\overline{m}_q - 1, \overline{m}_q\})$, with

$$r_{qi}^r = \begin{cases} 1, & \text{if arc } ([r, q, i], [r, q, i+1]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

r_{qi}^u : binary variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus \{\overline{m}_q\})$, with

$$r_{qi}^u = \begin{cases} 1, & \text{if arc } ([r, q, i], [u, q, i+1]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

r_{qi}^d : binary variable, $\forall (q, i) \in (F \setminus \{1\}) \times (I^q \setminus \{\overline{m}_q\})$, with

$$r_{qi}^d = \begin{cases} 1, & \text{if arc } ([r, q, i], [d, q, i+1]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

l_{qi}^l : binary variable, $\forall (q, i) \in F \times ((I^q \setminus \{\underline{m}_q, \underline{m}_q + 1\}) \cup \{(1, 2)\})$, with

$$l_{qi}^l = \begin{cases} 1, & \text{if arc } ([l, q, i], [l, q, i-1]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

l_{qi}^u : binary variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus \{m_q\})$, with

$$l_{qi}^u = \begin{cases} 1, & \text{if arc } ([l, q, i], [u, q, i-1]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

l_{qi}^d : binary variable, $\forall (q, i) \in (F \setminus \{1\}) \times (I^q \setminus \{m_q\})$, with

$$l_{qi}^d = \begin{cases} 1, & \text{if arc } ([l, q, i], [d, q, i-1]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

e_{qi1}^u : binary variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q)$, with

$$e_{qi1}^u = \begin{cases} 1, & \text{if arc } ([d, q+1, i], [u, q, i, 1]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

e_{qi1}^d : binary variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q)$, with

$$e_{qi1}^d = \begin{cases} 1, & \text{if arc } ([u, q, i], [d, q, i, 1]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

e_{qi2}^u : binary variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I_3^q \cup I_4^q)$, with

$$e_{qi2}^u = \begin{cases} 1, & \text{if arc } ([d, q+1, i], [u, q, i, 2]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

e_{qi2}^d : binary variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I_3^q \cup I_4^q)$, with

$$e_{qi2}^d = \begin{cases} 1, & \text{if arc } ([u, q, i], [d, q, i, 2]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

e_{qi3}^u : binary variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I_0^q \cup I_5^q)$, with

$$e_{qi3}^u = \begin{cases} 1, & \text{if arc } ([d, q+1, i], [u, q, i, 3]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

e_{qi3}^d : binary variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I_0^q \cup I_5^q)$, with

$$e_{qi3}^d = \begin{cases} 1, & \text{if arc } ([u, q, i], [d, q, i, 3]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

t_{qi}^u : binary variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q)$, with

$$t_{qi}^u = \begin{cases} 1, & \text{if arc } ([u, q, i], [u, q, i, 1]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

t_{qi}^d : binary variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q)$, with

$$t_{qi}^d = \begin{cases} 1, & \text{if arc } ([d, q+1, i], [d, q, i, 1]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

w_{qis}^u : binary variable, $\forall (q, i, s) \in B \times ((I_3^q \times \{1\}) \cup ((I_4^q \cup I_5^q) \times \{1, 2\}) \cup (I_0^q \times \{1, 2, 3\}))$, with

$$w_{qis}^u = \begin{cases} 1, & \text{if arc } ([u, q, i, s], [u, q, i, s+1]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

w_{qis}^d : binary variable, $\forall (q, i, s) \in B \times ((I_3^q \times \{1\}) \cup ((I_4^q \cup I_5^q) \times \{1, 2\}) \cup (I_0^q \times \{1, 2, 3\}))$, with

$$w_{qis}^d = \begin{cases} 1, & \text{if arc } ([d, q, i, s], [d, q, i, s+1]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

v_{qi}^u : binary variable, $\forall (q, i) \in (B \setminus \{p\}) \times I^q$, with

$$v_{qi}^u = \begin{cases} 1, & \text{if arc } ([u, q, i], [u, q+1, i]) \text{ for } i \in I_1^q, ([u, q, i, 1], [u, q+1, i]) \text{ for } i \in I_2^q, \\ & ([u, q, i, 2], [u, q+1, i]) \text{ for } i \in I_3^q, ([u, q, i, 3], [u, q+1, i]) \text{ for } i \in I_4^q \cup I_5^q \text{ or} \\ & ([u, q, i, 4], [u, q+1, i]) \text{ for } i \in I_0^q \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

$v_{qi}^{u,l}$: binary variable, $\forall (q, i) \in B \times (I^q \setminus \{\underline{m}_q\})$, with

$$v_{qi}^{u,l} = \begin{cases} 1, & \text{if arc } ([u, q, i], [l, q+1, i]) \text{ for } i \in I_1^q, ([u, q, i, 1], [l, q+1, i]) \text{ for } i \in I_2^q, \\ & ([u, q, i, 2], [l, q+1, i]) \text{ for } i \in I_3^q, ([u, q, i, 3], [l, q+1, i]) \text{ for } i \in I_4^q \cup I_5^q \text{ or} \\ & ([u, q, i, 4], [l, q+1, i]) \text{ for } i \in I_0^q \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

$v_{qi}^{u,r}$: binary variable, $\forall (q, i) \in B \times (I^q \setminus \{\bar{m}_q\})$, with

$$v_{qi}^{u,r} = \begin{cases} 1, & \text{if arc } ([u, q, i], [r, q+1, i]) \text{ for } i \in I_1^q, ([u, q, i, 1], [r, q+1, i]) \text{ for } i \in I_2^q, \\ & ([u, q, i, 2], [r, q+1, i]) \text{ for } i \in I_3^q, ([u, q, i, 3], [r, q+1, i]) \text{ for } i \in I_4^q \cup I_5^q \text{ or} \\ & ([u, q, i, 4], [r, q+1, i]) \text{ for } i \in I_0^q \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

v_{qi}^d : binary variable, $\forall (q, i) \in (B \setminus \{1\}) \times I^q$, with

$$v_{qi}^d = \begin{cases} 1, & \text{if arc } ([d, q+1, i], [d, q, i]) \text{ for } i \in I_1^q, ([d, q, i, 1], [d, q, i]) \text{ for } i \in I_2^q, \\ & ([d, q, i, 2], [d, q, i]) \text{ for } i \in I_3^q, ([d, q, i, 3], [d, q, i]) \text{ for } i \in I_4^q \cup I_5^q \text{ or} \\ & ([d, q, i, 4], [d, q, i]) \text{ for } i \in I_0^q \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

$v_{qi}^{d,l}$: binary variable, $\forall (q, i) \in B \times (I^q \setminus \{\underline{m}_q\}) \cup \{(1, 1)\}$, with

$$v_{qi}^{d,l} = \begin{cases} 1, & \text{if arc } ([u, q+1, i], [l, q, i]) \text{ for } i \in I_1^q, ([d, q, i, 1], [l, q, i]) \text{ for } i \in I_2^q, \\ & ([d, q, i, 2], [l, q, i]) \text{ for } i \in I_3^q, ([d, q, i, 3], [l, q, i]) \text{ for } i \in I_4^q \cup I_5^q \text{ or} \\ & ([d, q, i, 4], [l, q, i]) \text{ for } i \in I_0^q \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

$v_{qi}^{d,r}$: binary variable, $\forall (q, i) \in B \times (I^q \setminus \{\bar{m}_q\})$, with

$$v_{qi}^{d,r} = \begin{cases} 1, & \text{if arc } ([u, q+1, i], [r, q, i]) \text{ for } i \in I_1^q, ([u, q, i, 1], [r, q, i]) \text{ for } i \in I_2^q, \\ & ([u, q, i, 2], [r, q, i]) \text{ for } i \in I_3^q, ([u, q, i, 3], [r, q, i]) \text{ for } i \in I_4^q \cup I_5^q \text{ or} \\ & ([u, q, i, 4], [r, q, i]) \text{ for } i \in I_0^q \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

y_α^0 : binary variable, $\forall \alpha \in \{l, r, u\}$, with

$$y_\alpha^0 = \begin{cases} 1, & \text{if arc } ([0], [\alpha, 1, 1]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

y_0^l : binary variable, with

$$y_0^l = \begin{cases} 1, & \text{if arc } ([l, 1, 1], [0]) \text{ is contained in the tour} \\ 0, & \text{otherwise} \end{cases}$$

Real-valued variables to exclude subtours:

- \tilde{r}_{qi}^r : real-valued variable, $\forall (q, i) \in F \times (I^q \setminus \{\bar{m}_q - 1, \bar{m}_q\})$
- \tilde{r}_{qi}^u : real-valued variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus \{\bar{m}_q\})$
- \tilde{r}_{qi}^d : real-valued variable, $\forall (q, i) \in (F \setminus \{1\}) \times (I^q \setminus \{\bar{m}_q\})$
- \tilde{l}_{qi}^l : real-valued variable, $\forall (q, i) \in F \times ((I^q \setminus \{\underline{m}_q, \underline{m}_q + 1\}) \cup \{(1, 2)\})$
- \tilde{l}_{qi}^u : real-valued variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus \{\underline{m}_q\})$
- \tilde{l}_{qi}^d : real-valued variable, $\forall (q, i) \in (F \setminus \{1\}) \times (I^q \setminus \{\underline{m}_q\})$
- \tilde{e}_{qi1}^u : real-valued variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q)$
- \tilde{e}_{qi1}^d : real-valued variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q)$
- \tilde{e}_{qi2}^u : real-valued variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I_3^q \cup I_4^q)$
- \tilde{e}_{qi2}^d : real-valued variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I_3^q \cup I_5^q)$
- \tilde{e}_{qi3}^u : real-valued variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I_0^q \cup I_5^q)$
- \tilde{e}_{qi3}^d : real-valued variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I_0^q \cup I_4^q)$
- \tilde{r}_{qi}^u : real-valued variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q)$
- \tilde{r}_{qi}^d : real-valued variable, $\forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q)$
- \tilde{w}_{qis}^u : real-valued variable, $\forall (q, i, s) \in B \times ((I_3^q \times \{1\}) \cup ((I_4^q \cup I_5^q) \times \{1, 2\}) \cup (I_0^q \times \{1, 2, 3\}))$
- \tilde{w}_{qis}^d : real-valued variable, $\forall (q, i, s) \in B \times ((I_3^q \times \{1\}) \cup ((I_4^q \cup I_5^q) \times \{1, 2\}) \cup (I_0^q \times \{1, 2, 3\}))$
- \tilde{v}_{qi}^u : real-valued variable, $\forall (q, i) \in (B \setminus \{p\}) \times I^q$
- $\tilde{v}_{qi}^{u,l}$: real-valued variable, $\forall (q, i) \in B \times (I^q \setminus \{\underline{m}_q\})$
- $\tilde{v}_{qi}^{u,r}$: real-valued variable, $\forall (q, i) \in B \times (I^q \setminus \{\bar{m}_q\})$
- \tilde{v}_{qi}^d : real-valued variable, $\forall (q, i) \in (B \setminus \{1\}) \times I^q$
- $\tilde{v}_{qi}^{d,l}$: real-valued variable, $\forall (q, i) \in B \times (I^q \setminus \{\underline{m}_q\}) \cup \{(1, 1)\}$
- $\tilde{v}_{qi}^{d,r}$: real-valued variable, $\forall (q, i) \in B \times (I^q \setminus \{\bar{m}_q\})$
- \tilde{y}_α^0 : real-valued variable, $\forall \alpha \in \{l, r, u\}$
- \tilde{y}_0^l : real-valued variable

Constants:

- c_0 : distance between the depot and the intersection of cross aisle 1 with the first sub-aisle of block 1
- c^a : distance between two adjacent picking aisles
- \bar{c} : length of a sub-aisle
- $c_{qi}^{t,u}$: distance between cross aisle q and vertex $[u, q, i, 1]$, $\forall (q, i) \in B \times (I^q \setminus I_1^q)$
- $c_{qi}^{t,d}$: distance between cross aisle $q+1$ and vertex $[d, q, i, 1]$, $\forall (q, i) \in B \times (I^q \setminus I_1^q)$
- $c_{qis}^{e,u}$: distance between cross aisle $q+1$ and vertex $[u, q, i, s]$, $\forall (q, i, s) \in B \times (((I^q \setminus I_1^q) \times \{1\}) \cup ((I_3^q \cup I_4^q) \times \{2\}) \cup ((I_0^q \cup I_5^q) \times \{3\}))$
- $c_{qis}^{e,d}$: distance between cross aisle q and vertex $[d, q, i, s]$, $\forall (q, i, s) \in B \times (((I^q \setminus I_1^q) \times \{1\}) \cup ((I_3^q \cup I_5^q) \times \{2\}) \cup ((I_0^q \cup I_4^q) \times \{3\}))$
- $c_{qis}^{w,u}$: distance between vertex $[u, q, i, s]$ and vertex $[u, q, i, s+1]$, $\forall (q, i, s) \in B \times ((I_3^q \times \{1\}) \cup ((I_4^q \cup I_5^q) \times \{1, 2\}) \cup (I_0^q \times \{1, 2, 3\}))$

- $c_{qis}^{w,d}$: distance between vertex $[d, q, i, s]$ and vertex $[d, q, i, s + 1]$, $\forall (q, i, s) \in B \times ((I_3^q \times \{1\}) \cup ((I_4^q \cup I_5^q) \times \{1, 2\}) \cup (I_0^q \times \{1, 2, 3\}))$
- M : large number (e.g. number of vertices)

Objective Function:

$$\begin{aligned}
\min & \sum_{q=1}^{p+1} \sum_{i=\underline{m}_q}^{\bar{m}_q-2} c^a \cdot r_{qi}^r + \sum_{q=2}^{p+1} \sum_{i=\underline{m}_q}^{\bar{m}_q-1} c^a \cdot r_{qi}^d + \sum_{q=1}^p \sum_{i=\underline{m}_q}^{\bar{m}_q-1} c^a \cdot r_{qi}^u + \sum_{q=1}^p \sum_{\substack{i=\underline{m}_q-1, \\ i \geq \underline{m}_{q-1}}}^{\bar{m}_q-1} c^a \cdot r_{qi}^u + \sum_{q=1}^{p+1} \sum_{i=\underline{m}_q+2}^{\bar{m}_q} c^a \cdot l_{qi}^l + c^a \cdot l_{\underline{m}_1+1}^l \\
& + \sum_{q=2}^{p+1} \sum_{i=\underline{m}_q+1}^{\bar{m}_q} c^a \cdot l_{qi}^d + \sum_{q=1}^p \sum_{i=\underline{m}_q+1}^{\bar{m}_q} c^a \cdot l_{qi}^u + \sum_{q=1}^p \sum_{\substack{i=\bar{m}_q+1, \\ i \leq \bar{m}_{q-1}}}^{\bar{m}_q+1} c^a \cdot l_{qi}^u + \sum_{q=1}^{p-1} \sum_{\substack{i \in I_1^q, \\ i \leq \bar{m}_{q+1}}} \bar{c} \cdot v_{qi}^u + \sum_{q=1}^p \sum_{i \in I_1^q \setminus \{\bar{m}_q\}} \bar{c} \cdot v_{qi}^{u,r} \\
& + \sum_{q=1}^p \sum_{i \in I_1^q \setminus \{\bar{m}_q\}} \bar{c} \cdot v_{qi}^{u,l} + \sum_{q=1}^p \sum_{i \in I^q \setminus I_1^q} (c_{qi1}^{e,d} \cdot e_{qi1}^d + c_{qi1}^{e,u} \cdot e_{qi1}^u) + \sum_{q=1}^p \sum_{i \in I_3^q \cup I_5^q} c_{qi2}^{e,d} \cdot e_{qi2}^d + \sum_{q=1}^p \sum_{i \in I_0^q \cup I_4^q} c_{qi3}^{e,d} \cdot e_{qi3}^d \\
& + \sum_{q=1}^p \sum_{i \in I_3^q \cup I_4^q} c_{qi2}^{e,u} \cdot e_{qi2}^u + \sum_{q=1}^p \sum_{i \in I_0^q \cup I_5^q} c_{qi3}^{e,u} \cdot e_{qi3}^u + \sum_{q=1}^p \sum_{i \in I^q \setminus I_1^q} (c_{qi}^{t,d} \cdot t_{qi}^d + c_{qi}^{t,u} \cdot t_{qi}^u) + \sum_{q=1}^p \sum_{i \in I_0^q} \sum_{s=1}^3 (c_{qis}^{w,u} \cdot w_{qis}^u + c_{qis}^{w,d} \cdot w_{qis}^d) \\
& + \sum_{q=1}^p \sum_{i \in I_3^q} (c_{qi1}^{w,u} \cdot w_{qi1}^u + c_{qi1}^{w,d} \cdot w_{qi1}^d) + \sum_{q=1}^p \sum_{i \in I_4^q \cup I_5^q} \sum_{s=1}^2 (c_{qis}^{w,u} \cdot w_{qis}^u + c_{qis}^{w,d} \cdot w_{qis}^d) + \sum_{q=1}^{p-1} \sum_{\substack{i \in I_2^q, \\ i \leq \bar{m}_{q+1}}} c_{qi1}^{e,u} \cdot v_{qi}^u \\
& + \sum_{q=1}^{p-1} \sum_{\substack{i \in I^q \setminus (I_1^q \cup I_2^q): \\ i \leq \bar{m}_{q+1}}} c_{qi}^{t,d} \cdot v_{qi}^u + \sum_{q=2}^p \sum_{i \in I_2^q} c_{qi1}^{e,d} \cdot v_{qi}^d + \sum_{q=2}^p \sum_{i \in I^q \setminus (I_1^q \cup I_2^q)} c_{qi}^{t,u} \cdot v_{qi}^d + \sum_{q=1}^p \sum_{i \in I_2^q \setminus \{\bar{m}_q\}} c_{qi1}^{e,u} \cdot v_{qi}^{u,r} \\
& + \sum_{q=1}^p \sum_{i \in I^q \setminus (I_1^q \cup I_2^q \cup \{\bar{m}_q\})} c_{qi}^{t,d} \cdot v_{qi}^{u,r} + \sum_{q=1}^p \sum_{i \in I_2^q \setminus \{\bar{m}_{q-1}\}} c_{qi1}^{e,d} \cdot v_{qi}^{d,r} + \sum_{q=1}^p \sum_{i \in I^q \setminus (I_1^q \cup I_2^q \cup \{\bar{m}_{q-1}\})} c_{qi}^{t,u} \cdot v_{qi}^{d,r} + \sum_{q=1}^p \sum_{i \in I_2^q \setminus \{\bar{m}_q\}} c_{qi1}^{e,u} \cdot v_{qi}^{u,l} \\
& + \sum_{q=1}^p \sum_{i \in I^q \setminus (I_1^q \cup I_2^q \cup \{\bar{m}_q\})} c_{qi}^{t,d} \cdot v_{qi}^{u,l} + \sum_{q=1}^p \sum_{i \in I_2^q \setminus \{\bar{m}_{q-1}\}} c_{qi1}^{e,d} \cdot v_{qi}^{d,l} + \sum_{q=1}^p \sum_{\substack{i \in I^q \setminus (I_1^q \cup I_2^q \cup \{\bar{m}_{q-1}\}) \\ i \in I_2^q, \\ i=1}} c_{qi}^{t,u} \cdot v_{qi}^{d,l} + \sum_{\substack{i \in I_2^q \\ i=1}} c_{111}^{e,d} \cdot v_{11}^{d,l} \\
& + \sum_{\substack{i \in I^1 \setminus (I_1^1 \cup I_2^1): \\ i=1}} c_{111}^{t,u} \cdot v_{11}^{d,l} + c^0 \cdot (y_l^0 + y_r^0 + y_u^0 + y_0^l) + \sum_{q=2}^p \sum_{i \in I_1^q} \bar{c} \cdot v_{qi}^d + \sum_{q=1}^p \sum_{\substack{i \in I_1^q: \\ i < \bar{m}_{q-1}}} \bar{c} \cdot v_{qi}^{d,r} + \sum_{q=1}^p \sum_{\substack{i \in I_1^q: \\ i > \bar{m}_{q-1}}} \bar{c} \cdot v_{qi}^{d,l} + \sum_{\substack{i \in I_1^q: \\ i=1}} \bar{c} \cdot v_{11}^{d,l} \quad (6)
\end{aligned}$$

Depot Inclusion Constraint:

$$y_l^0 + y_r^0 + y_u^0 \geq 1 \quad (7)$$

Item Inclusion Constraints:

$$w_{qi1}^u + w_{qi3}^d \geq 1 \quad \forall q \in B, i \in I_0^q \quad (8)$$

$$w_{qi3}^u + w_{qi1}^d \geq 1 \quad \forall q \in B, i \in I_0^q \quad (9)$$

$$t_{qi}^u + e_{qi1}^d + t_{qi}^d + e_{qi1}^u \geq 1 \quad \forall q \in B, i \in I_2^q \quad (10)$$

$$w_{qi1}^u + w_{qi1}^d + e_{qi2}^d \geq 1 \quad \forall q \in B, i \in I_3^q \quad (11)$$

$$w_{qi1}^u + w_{qi1}^d + e_{qi2}^u \geq 1 \quad \forall q \in B, i \in I_3^q \quad (12)$$

$$w_{qi1}^u + w_{qi2}^d + e_{qi3}^d \geq 1 \quad \forall q \in B, i \in I_4^q \quad (13)$$

$$w_{qi2}^u + w_{qi1}^d \geq 1 \quad \forall q \in B, i \in I_4^q \quad (14)$$

$$w_{qi1}^u + w_{qi2}^d \geq 1 \quad \forall q \in B, i \in I_5^q \quad (15)$$

$$w_{qi2}^u + e_{qi3}^u + w_{qi1}^d \geq 1 \quad \forall q \in B, i \in I_5^q \quad (16)$$

Degree Constraints:

- Constraint corresponding to the depot

$$y_l^0 + y_r^0 + y_u^0 = y_0^l \quad (17)$$

- Constraints corresponding to vertices $[r, q, i]$

$$r_{1i}^r + r_{1i}^u = r_{1,i-1}^r + v_{1i}^{d,r} \quad \forall i \in I^1 \setminus \{\underline{m}_1, \bar{m}_1 - 1, \bar{m}_1\} \quad (18)$$

$$r_{1,\underline{m}_1}^r + r_{1,\underline{m}_1}^u = y_r^0 + v_{1,\underline{m}_1}^{d,r} \quad (19)$$

$$r_{1,\bar{m}_1-1}^u = r_{1,\bar{m}_1-1}^r + v_{1,\bar{m}_1-1}^{d,r} \quad (20)$$

$$r_{qi}^r + r_{qi}^u + r_{qi}^d = r_{q,i-1}^r + v_{q-1,i}^{u,r} + v_{qi}^{d,r} \quad \forall q \in F \setminus \{1, p+1\}, i \in I^q \setminus \{\underline{m}_{q-1}, \bar{m}_{q-1} - 1, \bar{m}_{q-1}\} \text{ with } \underline{m}_q - 1 < i < \bar{m}_q \quad (21)$$

$$r_{qi}^r + r_{qi}^d = r_{q,i-1}^r + v_{q-1,i}^{u,r} \quad \forall q \in F \setminus \{1, p+1\}, i \in I^q \setminus \{\underline{m}_{q-1}, \bar{m}_{q-1} - 1, \bar{m}_{q-1}\} \text{ with } i < \underline{m}_q - 1 \text{ or } i > \bar{m}_q \quad (22)$$

$$r_{qi}^r + r_{qi}^d = r_{q,i-1}^r + v_{q-1,i}^{u,r} + v_{qi}^{d,r} \quad \forall q \in F \setminus \{1, p+1\}, i \in I^q \setminus \{\underline{m}_{q-1}, \bar{m}_{q-1} - 1, \bar{m}_{q-1}\} \text{ with } i = \bar{m}_q \quad (23)$$

$$r_{qi}^r + r_{qi}^u + r_{qi}^d = r_{q,i-1}^r + v_{q-1,i}^{u,r} \quad \forall q \in F \setminus \{1, p+1\}, i \in I^q \setminus \{\underline{m}_{q-1}, \bar{m}_{q-1} - 1, \bar{m}_{q-1}\} \text{ with } i = \underline{m}_q - 1 \quad (24)$$

$$r_{q,\underline{m}_{q-1}}^r + r_{q,\underline{m}_{q-1}}^u + r_{q,\underline{m}_{q-1}}^d = v_{q-1,\underline{m}_{q-1}}^{u,r} + v_{q,\underline{m}_{q-1}}^{d,r} \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \underline{m}_q = \underline{m}_{q-1} \quad (25)$$

$$r_{q,\underline{m}_{q-1}}^r + r_{q,\underline{m}_{q-1}}^u + r_{q,\underline{m}_{q-1}}^d = v_{q-1,\underline{m}_{q-1}}^{u,r} \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \underline{m}_q = \underline{m}_{q-1} + 1 \quad (26)$$

$$r_{q,\underline{m}_{q-1}}^r + r_{q,\underline{m}_{q-1}}^d = v_{q-1,\underline{m}_{q-1}}^{u,r} \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \underline{m}_q > \underline{m}_{q-1} + 1 \quad (27)$$

$$r_{q,\bar{m}_{q-1}-1}^u + r_{q,\bar{m}_{q-1}-1}^d = r_{q,\bar{m}_{q-1}-2}^r + v_{q-1,\bar{m}_{q-1}-1}^{u,r} + v_{q,\bar{m}_{q-1}-1}^{d,r} \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \bar{m}_q = \bar{m}_{q-1} \quad (28)$$

$$r_{q,\bar{m}_{q-1}-1}^d = r_{q,\bar{m}_{q-1}-2}^r + v_{q-1,\bar{m}_{q-1}-1}^{u,r} + v_{q,\bar{m}_{q-1}-1}^{d,r} \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \bar{m}_q = \bar{m}_{q-1} - 1 \quad (29)$$

$$r_{q,\bar{m}_{q-1}-1}^d = r_{q,\bar{m}_{q-1}-2}^r + v_{q-1,\bar{m}_{q-1}-1}^{u,r} \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \bar{m}_q < \bar{m}_{q-1} - 1 \quad (30)$$

$$r_{p+1,i}^r + r_{p+1,i}^d = r_{p+1,i-1}^r + v_{pi}^{u,r} \quad \forall i \in I^p \setminus \{\underline{m}_p, \bar{m}_p - 1, \bar{m}_p\} \quad (31)$$

$$r_{p+1,\underline{m}_p}^r + r_{p+1,\underline{m}_p}^d = v_{p,\underline{m}_p}^{u,r} \quad (32)$$

$$r_{p+1,\bar{m}_p-1}^d = r_{p+1,\bar{m}_p-2}^r + v_{p,\bar{m}_p-1}^{u,r} \quad (33)$$

- Constraints corresponding to vertices $[l, q, i]$

$$l_{1i}^l + l_{1i}^u = l_{1,i+1}^l + v_{1i}^{d,l} \quad \forall i \in I^1 \setminus \{\underline{m}_1, \bar{m}_1\} \quad (34)$$

$$y_0^l = y_l^0 + l_{1,\underline{m}_1+1}^l + v_{1,\underline{m}_1}^{d,l} \quad (35)$$

$$l_{1,\bar{m}_1}^l + l_{1,\bar{m}_1}^u = v_{1,\bar{m}_1}^{d,l} \quad (36)$$

$$l_{qi}^l + l_{qi}^u + l_{qi}^d = l_{q,i+1}^l + v_{qi}^{d,l} + v_{q-1,i}^{u,l} \quad \forall q \in F \setminus \{1, p+1\}, i \in I^q \setminus \{\underline{m}_{q-1}, \underline{m}_{q-1} + 1, \bar{m}_{q-1}\} \text{ with } \underline{m}_q < i < \bar{m}_q + 1 \quad (37)$$

$$l_{qi}^l + l_{qi}^d = l_{q,i+1}^l + v_{q-1,i}^{u,l} \quad \forall q \in F \setminus \{1, p+1\}, i \in I^q \setminus \{\underline{m}_{q-1}, \underline{m}_{q-1} + 1, \bar{m}_{q-1}\} \text{ with } i < \underline{m}_q \text{ or } i > \bar{m}_q + 1 \quad (38)$$

$$l_{qi}^l + l_{qi}^d = l_{q,i+1}^l + v_{qi}^{d,l} + v_{q-1,i}^{u,l} \quad \forall q \in F \setminus \{1, p+1\}, i \in I^q \setminus \{\underline{m}_{q-1}, \underline{m}_{q-1} + 1, \bar{m}_{q-1}\} \text{ with } i = \underline{m}_q \quad (39)$$

$$l_{qi}^l + l_{qi}^u + l_{qi}^d = l_{q,i+1}^l + v_{q-1,i}^{u,l} \quad \forall q \in F \setminus \{1, p+1\}, i \in I^q \setminus \{\underline{m}_{q-1}, \underline{m}_{q-1} + 1, \bar{m}_{q-1}\} \text{ with } i = \bar{m}_q + 1 \quad (40)$$

$$l_{q,\underline{m}_{q-1}+1}^u + l_{q,\underline{m}_{q-1}+1}^d = l_{q,\underline{m}_{q-1}+2}^l + v_{q,\underline{m}_{q-1}+1}^{d,l} + v_{q-1,\underline{m}_{q-1}+1}^{u,l} \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \underline{m}_q = \underline{m}_{q-1} \quad (41)$$

$$l_{q,\underline{m}_{q-1}+1}^d = l_{q,\underline{m}_{q-1}+2}^l + v_{q,\underline{m}_{q-1}+1}^{d,l} + v_{q-1,\underline{m}_{q-1}+1}^{u,l} \quad (42)$$

$$l_{q,\underline{m}_{q-1}+1}^d = l_{q,\underline{m}_{q-1}+2}^l + v_{q-1,\underline{m}_{q-1}+1}^{u,l} \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \underline{m}_q > \underline{m}_{q-1} + 1 \quad (43)$$

$$l_{q,\bar{m}_{q-1}}^l + l_{q,\bar{m}_{q-1}}^u + l_{q,\bar{m}_{q-1}}^d = v_{q,\bar{m}_{q-1}}^{d,l} + v_{q-1,\bar{m}_{q-1}}^{u,l} \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \bar{m}_q = \bar{m}_{q-1} \quad (44)$$

$$l_{q,\bar{m}_{q-1}}^l + l_{q,\bar{m}_{q-1}}^u + l_{q,\bar{m}_{q-1}}^d = v_{q-1,\bar{m}_{q-1}}^{u,l} \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \bar{m}_q = \bar{m}_{q-1} - 1 \quad (45)$$

$$l_{q,\bar{m}_{q-1}}^l + l_{q,\bar{m}_{q-1}}^d = v_{q-1,\bar{m}_{q-1}}^{u,l} \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \bar{m}_q < \bar{m}_{q-1} - 1 \quad (46)$$

$$l_{p+1,i}^l + l_{p+1,i}^d = l_{p+1,i+1}^l + v_{pi}^{u,l} \quad \forall i \in I^p \setminus \{\underline{m}_p, \underline{m}_p + 1, \bar{m}_p\} \quad (47)$$

$$l_{p+1,\underline{m}_p+1}^d = l_{p+1,\underline{m}_p+2}^l + v_{p,\underline{m}_p+1}^{u,l} \quad (48)$$

$$l_{p+1,\bar{m}_p}^l + l_{p+1,\bar{m}_p}^d = v_{p,\bar{m}_p}^{u,l} \quad (49)$$

- Constraints corresponding to vertices $[u, q, i]$

$$v_{qi}^u + v_{qi}^{u,l} + v_{qi}^{u,r} = l_{q,i+1}^u + r_{q,i-1}^u + v_{q-1,i}^u \quad \forall q \in B \setminus \{1, p\}, i \in I_1^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} \quad (50)$$

$$v_{1,\underline{m}_1}^u + v_{1,\underline{m}_1}^{u,r} = l_{1,\underline{m}_1+1}^u + y_u^0 \quad \text{if } \underline{m}_1 = \underline{m}_2 \text{ and } \underline{m}_1 \in I_1^1 \quad (51)$$

$$v_{q,\bar{m}_q}^u + v_{q,\bar{m}_q}^{u,l} = r_{q,\bar{m}_q-1}^u + v_{q-1,\bar{m}_q}^u \quad \forall q \in B \setminus \{1, p\} \text{ with } \bar{m}_q \in I_1^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \text{ and } \bar{m}_q = \bar{m}_{q-1} \quad (52)$$

$$v_{q,\bar{m}_q}^u + v_{q,\bar{m}_q}^{u,l} = r_{q,\bar{m}_q-1}^u + l_{q,\bar{m}_q+1}^u + v_{q-1,\bar{m}_q}^u \quad \forall q \in B \setminus \{1, p\} \text{ with } \bar{m}_q \in I_1^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \text{ and } \bar{m}_q \neq \bar{m}_{q-1} \quad (53)$$

$$v_{qi}^{u,l} + v_{qi}^{u,r} = l_{q,i+1}^u + r_{q,i-1}^u + v_{q-1,i}^u \quad \forall q \in B \setminus \{1, p\}, i \in I_1^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (54)$$

$$v_{1,\underline{m}_1}^{u,r} = l_{1,\underline{m}_1+1}^u + y_u^0 \quad \text{if } \underline{m}_1 \neq \underline{m}_2 \text{ and } \underline{m}_1 \in I_1^1 \quad (55)$$

$$v_{q,\bar{m}_q}^{u,l} = r_{q,\bar{m}_q-1}^u + v_{q-1,\bar{m}_q}^u \quad \forall q \in B \setminus \{1, p\} \text{ with } \bar{m}_q \in I_1^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \text{ and } \bar{m}_q = \bar{m}_{q-1} \quad (56)$$

$$v_{q,\bar{m}_q}^{u,l} = r_{q,\bar{m}_q-1}^u + l_{q,\bar{m}_q+1}^u + v_{q-1,\bar{m}_q}^u \quad \forall q \in B \setminus \{1, p\} \text{ with } \bar{m}_q \in I_1^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \text{ and } \bar{m}_q \neq \bar{m}_{q-1} \quad (57)$$

$$v_{q,\underline{m}_q}^u + v_{q,\underline{m}_q}^{u,r} = l_{q,\underline{m}_q+1}^u + v_{q-1,\underline{m}_q}^u \quad \forall q \in B \setminus \{1, p\} \text{ with } \underline{m}_q \in I_1^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \text{ and } \underline{m}_q = \underline{m}_{q-1} \quad (58)$$

$$v_{q,\underline{m}_q}^u + v_{q,\underline{m}_q}^{u,r} = r_{q,\underline{m}_q-1}^u + l_{q,\underline{m}_q+1}^u + v_{q-1,\underline{m}_q}^u \quad \forall q \in B \setminus \{1, p\} \text{ with } \underline{m}_q \in I_1^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \text{ and } \underline{m}_q \neq \underline{m}_{q-1} \quad (59)$$

$$v_{q,\underline{m}_q}^{u,r} = l_{q,\underline{m}_q+1}^u + v_{q-1,\underline{m}_q}^u \quad \forall q \in B \setminus \{1, p\} \text{ with } \underline{m}_q \in I_1^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} \text{ and } \underline{m}_q = \underline{m}_{q-1} \quad (60)$$

$$v_{q,\underline{m}_q}^{u,r} = r_{q,\underline{m}_q-1}^u + l_{q,\underline{m}_q+1}^u + v_{q-1,\underline{m}_q}^u \quad \forall q \in B \setminus \{1, p\} \text{ with } \underline{m}_q \in I_1^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} \text{ and } \underline{m}_q \neq \underline{m}_{q-1} \quad (61)$$

$$v_{pi}^{u,l} + v_{pi}^{u,r} = l_{p,i+1}^u + r_{p,i-1}^u + v_{p-1,i}^u \quad \forall i \in I_1^p \setminus \{\underline{m}_p, \bar{m}_p\} \quad (62)$$

$$v_{p,\underline{m}_p}^{u,r} = l_{p,\underline{m}_p+1}^u + v_{p-1,\underline{m}_p}^u \quad \text{if } \underline{m}_p \in I_1^p \quad (63)$$

$$v_{p,\bar{m}_p}^{u,l} = r_{p,\bar{m}_p-1}^u + v_{p-1,\bar{m}_p}^u \quad \text{if } \bar{m}_p \in I_1^p \quad (64)$$

$$v_{1i}^u + v_{1i}^{u,l} + v_{1i}^{u,r} = l_{1,i+1}^u + r_{1,i-1}^u \quad \forall i \in I_1^1 \setminus \{\underline{m}_1, \bar{m}_1\} \text{ with } \underline{m}_2 \leq i \leq \bar{m}_2 \quad (65)$$

$$v_{1i}^{u,l} + v_{1i}^{u,r} = l_{1,i+1}^u + r_{1,i-1}^u \quad \forall i \in I_1^1 \setminus \{\underline{m}_1, \bar{m}_1\} \text{ with } i < \underline{m}_2 \text{ or } i > \bar{m}_2 \quad (66)$$

$$v_{1,\bar{m}_1}^u + v_{1,\bar{m}_1}^{u,l} = r_{1,\bar{m}_1-1}^u \quad \text{if } \bar{m}_1 = \bar{m}_2 \text{ and } \bar{m}_1 \in I_1^1 \quad (67)$$

$$v_{1,\bar{m}_1}^{u,l} = r_{1,\bar{m}_1-1}^u \quad \text{if } \bar{m}_1 \neq \bar{m}_2 \text{ and } \bar{m}_1 \in I_1^1 \quad (68)$$

$$t_{1i}^u + e_{1i1}^d = l_{1,i+1}^u + r_{1,i-1}^u \quad \forall i \in I_2^1 \setminus \{\underline{m}_1, \bar{m}_1\} \quad (69)$$

$$t_{1,\underline{m}_1}^u + e_{1,\underline{m}_1,1}^d = l_{1,\underline{m}_1+1}^u + y_u^0 \quad \text{if } \underline{m}_1 \in I_2^1 \quad (70)$$

$$t_{1,\bar{m}_1}^u + e_{1,\bar{m}_1,1}^d = r_{1,\bar{m}_1-1}^u \quad \text{if } \bar{m}_1 \in I_2^1 \quad (71)$$

$$t_{qi}^u + e_{qi1}^d = l_{q,i+1}^u + r_{q,i-1}^u + v_{q-1,i}^u \quad \forall q \in B \setminus \{1\}, i \in I_2^q \setminus \{\underline{m}_q, \bar{m}_q\} \quad (72)$$

$$t_{q,\underline{m}_q}^u + e_{q,\underline{m}_q,1}^d = l_{q,\underline{m}_q+1}^u + v_{q-1,\underline{m}_q}^u \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_2^q \text{ and } \underline{m}_q = \underline{m}_{q-1} \quad (73)$$

$$t_{q,\underline{m}_q}^u + e_{q,\underline{m}_q,1}^d = r_{q,\underline{m}_q-1}^u + l_{q,\underline{m}_q+1}^u + v_{q-1,\underline{m}_q}^u \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_2^q \text{ and } \underline{m}_q \neq \underline{m}_{q-1} \quad (74)$$

$$t_{q,\bar{m}_q}^u + e_{q,\bar{m}_q,1}^d = r_{q,\bar{m}_q-1}^u + v_{q-1,\bar{m}_q}^u \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_2^q \text{ and } \bar{m}_q = \bar{m}_{q-1} \quad (75)$$

$$t_{q,\bar{m}_q}^u + e_{q,\bar{m}_q,1}^d = r_{q,\bar{m}_q-1}^u + l_{q,\bar{m}_q+1}^u + v_{q-1,\bar{m}_q}^u \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_2^q \text{ and } \bar{m}_q \neq \bar{m}_{q-1} \quad (76)$$

$$t_{1i}^u + e_{1i1}^d + e_{1i3}^d = l_{1,i+1}^u + r_{1,i-1}^u \quad \forall i \in (I_0^1 \cup I_4^1) \setminus \{\underline{m}_1, \bar{m}_1\} \quad (77)$$

$$t_{1,\underline{m}_1}^u + e_{1,\underline{m}_1,1}^d + e_{1,\underline{m}_1,3}^d = l_{1,\underline{m}_1+1}^u + y_u^0 \quad \text{if } \underline{m}_1 \in I_0^1 \cup I_4^1 \quad (78)$$

$$t_{1,\bar{m}_1}^u + e_{1,\bar{m}_1,1}^d + e_{1,\bar{m}_1,3}^d = r_{1,\bar{m}_1-1}^u \quad \text{if } \bar{m}_1 \in I_0^1 \cup I_4^1 \quad (79)$$

$$t_{qi}^u + e_{qi1}^d + e_{qi3}^d = l_{q,i+1}^u + r_{q,i-1}^u + v_{q-1,i}^u \quad \forall q \in B \setminus \{1\}, i \in (I_2^q \cup I_4^q) \setminus \{\underline{m}_q, \bar{m}_q\} \quad (80)$$

$$t_{q,\underline{m}_q}^u + e_{q,\underline{m}_q,1}^d + e_{q,\underline{m}_q,3}^d = l_{q,\underline{m}_q+1}^u + v_{q-1,\underline{m}_q}^u \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_0^q \cup I_4^q \text{ and } \underline{m}_q = \underline{m}_{q-1} \quad (81)$$

$$t_{q,\underline{m}_q}^u + e_{q,\underline{m}_q,1}^d + e_{q,\underline{m}_q,3}^d = r_{q,\underline{m}_q-1}^u + l_{q,\underline{m}_q+1}^u + v_{q-1,\underline{m}_q}^u \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_0^q \cup I_4^q \text{ and } \underline{m}_q \neq \underline{m}_{q-1} \quad (82)$$

$$t_{q,\bar{m}_q}^u + e_{q,\bar{m}_q,1}^d + e_{q,\bar{m}_q,3}^d = r_{q,\bar{m}_q-1}^u + v_{q-1,\bar{m}_q}^u \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_0^q \cup I_4^q \text{ and } \bar{m}_q = \bar{m}_{q-1} \quad (83)$$

$$t_{q,\bar{m}_q}^u + e_{q,\bar{m}_q,1}^d + e_{q,\bar{m}_q,3}^d = r_{q,\bar{m}_q-1}^u + l_{q,\bar{m}_q+1}^u + v_{q-1,\bar{m}_q}^u \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_0^q \cup I_4^q \text{ and } \bar{m}_q \neq \bar{m}_{q-1} \quad (84)$$

$$t_{1i}^u + e_{1i1}^d + e_{1i2}^d = l_{1,i+1}^u + r_{1,i-1}^u \quad \forall i \in (I_3^1 \cup I_5^1) \setminus \{\underline{m}_1, \bar{m}_1\} \quad (85)$$

$$t_{1,\underline{m}_1}^u + e_{1,\underline{m}_1,1}^d + e_{1,\underline{m}_1,2}^d = l_{1,\underline{m}_1+1}^u + y_u^0 \quad \text{if } \underline{m}_1 \in I_3^1 \cup I_5^1 \quad (86)$$

$$t_{1,\bar{m}_1}^u + e_{1,\bar{m}_1,1}^d + e_{1,\bar{m}_1,2}^d = r_{1,\bar{m}_1-1}^u \quad \text{if } \bar{m}_1 \in I_3^1 \cup I_5^1 \quad (87)$$

$$t_{qi}^u + e_{qi1}^d + e_{qi2}^d = l_{q,i+1}^u + r_{q,i-1}^u + v_{q-1,i}^u \quad \forall q \in B \setminus \{1\}, i \in (I_3^q \cup I_5^q) \setminus \{\underline{m}_q, \bar{m}_q\} \quad (88)$$

$$t_{q,\underline{m}_q}^u + e_{q,\underline{m}_q,1}^d + e_{q,\underline{m}_q,2}^d = l_{q,\underline{m}_q+1}^u + v_{q-1,\underline{m}_q}^u \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_3^q \cup I_5^q \text{ and } \underline{m}_q = \underline{m}_{q-1} \quad (89)$$

$$t_{q,\underline{m}_q}^u + e_{q,\underline{m}_q,1}^d + e_{q,\underline{m}_q,2}^d = r_{q,\underline{m}_q-1}^u + l_{q,\underline{m}_q+1}^u + v_{q-1,\underline{m}_q}^u \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_3^q \cup I_5^q \text{ and } \underline{m}_q \neq \underline{m}_{q-1} \quad (90)$$

$$t_{q,\bar{m}_q}^u + e_{q,\bar{m}_q,1}^d + e_{q,\bar{m}_q,2}^d = r_{q,\bar{m}_q-1}^u + v_{q-1,\bar{m}_q}^u \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_3^q \cup I_5^q \text{ and } \bar{m}_q = \bar{m}_{q-1} \quad (91)$$

$$t_{q,\bar{m}_q}^u + e_{q,\bar{m}_q,1}^d + e_{q,\bar{m}_q,2}^d = r_{q,\bar{m}_q-1}^u + l_{q,\bar{m}_q+1}^u + v_{q-1,\bar{m}_q}^u \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_3^q \cup I_5^q \text{ and } \bar{m}_q \neq \bar{m}_{q-1} \quad (92)$$

- Constraints corresponding to vertices $[d, q, i]$

$$v_{1i}^{d,l} + v_{1i}^{d,r} = l_{2,i+1}^d + r_{2,i-1}^d + v_{2i}^d \quad \forall i \in I_1^1 \setminus \{\underline{m}_1, \bar{m}_1\} \text{ with } \underline{m}_2 \leq i \leq \bar{m}_2 \quad (93)$$

$$v_{1i}^{d,l} + v_{1i}^{d,r} = l_{2,i+1}^d + r_{2,i-1}^d \quad \forall i \in I_1^1 \setminus \{\underline{m}_1, \bar{m}_1\} \text{ with } i < \underline{m}_2 \text{ or } i > \bar{m}_2 \quad (94)$$

$$v_{1,\underline{m}_1}^{d,l} + v_{1,\underline{m}_1}^{d,r} = l_{2,\underline{m}_1+1}^d + v_{2,\underline{m}_1}^d \quad \text{if } \underline{m}_1 \in I_1^1 \text{ and } \underline{m}_1 = \underline{m}_2 \quad (95)$$

$$v_{1,\underline{m}_1}^{d,l} + v_{1,\underline{m}_1}^{d,r} = l_{2,\underline{m}_1+1}^d \quad \text{if } \underline{m}_1 \in I_1^1 \text{ and } \underline{m}_1 \neq \underline{m}_2 \quad (96)$$

$$v_{1,\bar{m}_1}^{d,l} = r_{2,\bar{m}_1-1}^d + v_{2,\bar{m}_1}^d \quad \text{if } \bar{m}_1 \in I_1^1 \text{ and } \bar{m}_1 = \bar{m}_2 \quad (97)$$

$$v_{1,\bar{m}_1}^{d,l} = r_{2,\bar{m}_1-1}^d \quad \text{if } \bar{m}_1 \in I_1^1 \text{ and } \bar{m}_1 \neq \bar{m}_2 \quad (98)$$

$$v_{qi}^{d,l} + v_{qi}^{d,r} + v_{qi}^d = l_{q+1,i+1}^d + r_{q+1,i-1}^d + v_{q+1,i}^d \quad \forall q \in B \setminus \{1, p\}, i \in I_1^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} \quad (99)$$

$$\begin{aligned}
v_{qi}^{d,l} + v_{qi}^{d,r} + v_{qi}^d &= l_{q+1,i+1}^d + r_{q+1,i-1}^d & \forall q \in B \setminus \{1, p\}, i \in I_1^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} & (100) \\
v_{q,\underline{m}_q}^{d,r} + v_{q,\underline{m}_q}^d &= l_{q+1,\underline{m}_q+1}^d + v_{q+1,\underline{m}_q}^d & \forall q \in B \setminus \{1, p\} \text{ with } \underline{m}_q \in I_1^q \text{ and } \underline{m}_q = \underline{m}_{q+1}, \underline{m}_q = \underline{m}_{q-1} & (101) \\
v_{q,\underline{m}_q}^{d,r} + v_{q,\underline{m}_q}^d &= l_{q+1,\underline{m}_q+1}^d & \forall q \in B \setminus \{1, p\} \text{ with } \underline{m}_q \in I_1^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1}, \underline{m}_q = \underline{m}_{q-1} & (102) \\
v_{q,\bar{m}_q}^{d,l} + v_{q,\bar{m}_q}^d &= r_{q+1,\bar{m}_q-1}^d + v_{q+1,\bar{m}_q}^d & \forall q \in B \setminus \{1, p\} \text{ with } \bar{m}_q \in I_1^q \text{ and } \bar{m}_q = \bar{m}_{q+1}, \bar{m}_q = \bar{m}_{q-1} & (103) \\
v_{q,\bar{m}_q}^{d,l} + v_{q,\bar{m}_q}^d &= r_{q+1,\bar{m}_q-1}^d & \forall q \in B \setminus \{1, p\} \text{ with } \bar{m}_q \in I_1^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1}, \bar{m}_q = \bar{m}_{q-1} & (104) \\
v_{pi}^{d,l} + v_{pi}^{d,r} + v_{pi}^d &= l_{p+1,i+1}^d + r_{p+1,i-1}^d & \forall i \in I_1^p \setminus \{\underline{m}_p, \bar{m}_p\} & (105) \\
v_{p,\underline{m}_p}^{d,l} + v_{p,\underline{m}_p}^{d,r} + v_{p,\underline{m}_p}^d &= l_{p+1,\underline{m}_p+1}^d & \text{if } \underline{m}_p \in I_1^p \text{ and } \underline{m}_p \neq \underline{m}_{p-1} & (106) \\
v_{p,\underline{m}_p}^{d,r} + v_{p,\underline{m}_p}^d &= l_{p+1,\underline{m}_p+1}^d & \text{if } \underline{m}_p \in I_1^p \text{ and } \underline{m}_p = \underline{m}_{p-1} & (107) \\
v_{p,\bar{m}_p}^{d,l} + v_{p,\bar{m}_p}^{d,r} + v_{p,\bar{m}_p}^d &= r_{p+1,\bar{m}_p-1}^d & \text{if } \bar{m}_p \in I_1^p \text{ and } \bar{m}_p \neq \bar{m}_{p-1} & (108) \\
v_{p,\bar{m}_p}^{d,l} + v_{p,\bar{m}_p}^d &= r_{p+1,\bar{m}_p-1}^d & \text{if } \bar{m}_p \in I_1^p \text{ and } \bar{m}_p = \bar{m}_{p-1} & (109) \\
v_{q,\underline{m}_q}^{d,l} + v_{q,\underline{m}_q}^{d,r} + v_{q,\underline{m}_q}^d &= l_{q+1,\underline{m}_q+1}^d + v_{q+1,\underline{m}_q}^d & \forall q \in B \setminus \{1, p\} \text{ with } \underline{m}_q \in I_1^q \text{ and } \underline{m}_q = \underline{m}_{q+1}, \underline{m}_q \neq \underline{m}_{q-1} & (110) \\
v_{q,\underline{m}_q}^{d,l} + v_{q,\underline{m}_q}^{d,r} + v_{q,\underline{m}_q}^d &= l_{q+1,\underline{m}_q+1}^d & \forall q \in B \setminus \{1, p\} \text{ with } \underline{m}_q \in I_1^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1}, \underline{m}_q \neq \underline{m}_{q-1} & (111) \\
v_{q,\bar{m}_q}^{d,l} + v_{q,\bar{m}_q}^{d,r} + v_{q,\bar{m}_q}^d &= r_{q+1,\bar{m}_q-1}^d + v_{q+1,\bar{m}_q}^d & \forall q \in B \setminus \{1, p\} \text{ with } \bar{m}_q \in I_1^q \text{ and } \bar{m}_q = \bar{m}_{q+1}, \bar{m}_q \neq \bar{m}_{q-1} & (112) \\
v_{q,\bar{m}_q}^{d,l} + v_{q,\bar{m}_q}^{d,r} + v_{q,\bar{m}_q}^d &= r_{q+1,\bar{m}_q-1}^d & \forall q \in B \setminus \{1, p\} \text{ with } \bar{m}_q \in I_1^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1}, \bar{m}_q \neq \bar{m}_{q-1} & (113) \\
e_{qi1}^u + t_{qi}^d &= l_{q+1,i+1}^d + r_{q+1,i-1}^d + v_{q+1,i}^d & \forall q \in B \setminus \{p\} \text{ with } i \in I_2^q \setminus \{\underline{m}_q, \bar{m}_q\}, \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} & (114) \\
e_{qi1}^u + t_{qi}^d &= l_{q+1,i+1}^d + r_{q+1,i-1}^d & \forall q \in B \setminus \{p\} \text{ with } i \in I_2^q \setminus \{\underline{m}_q, \bar{m}_q\}, i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} & (115) \\
e_{q,\underline{m}_q,1}^u + t_{q,\underline{m}_q}^d &= l_{q+1,\underline{m}_q+1}^d + v_{q+1,\underline{m}_q}^d & \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_2^q \text{ and } \underline{m}_q = \underline{m}_{q+1} & (116) \\
e_{q,\underline{m}_q,1}^u + t_{q,\underline{m}_q}^d &= l_{q+1,\underline{m}_q+1}^d & \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_2^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} & (117) \\
e_{q,\bar{m}_q,1}^u + t_{q,\bar{m}_q}^d &= r_{q+1,\bar{m}_q-1}^d + v_{q+1,\bar{m}_q}^d & \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_2^q \text{ and } \bar{m}_q = \bar{m}_{q+1} & (118) \\
e_{q,\bar{m}_q,1}^u + t_{q,\bar{m}_q}^d &= r_{q+1,\bar{m}_q-1}^d & \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_2^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} & (119) \\
e_{pi1}^u + t_{pi}^d &= l_{p+1,i+1}^d + r_{p+1,i-1}^d & \forall i \in I_2^p \setminus \{\underline{m}_p, \bar{m}_p\} & (120) \\
e_{p,\underline{m}_p,1}^u + t_{p,\underline{m}_p}^d &= l_{p+1,\underline{m}_p+1}^d & \text{if } \underline{m}_p \in I_2^p & (121) \\
e_{p,\bar{m}_p,1}^u + t_{p,\bar{m}_p}^d &= r_{p+1,\bar{m}_p-1}^d & \text{if } \bar{m}_p \in I_2^p & (122) \\
e_{qi1}^u + e_{qi2}^u + t_{qi}^d &= l_{q+1,i+1}^d + r_{q+1,i-1}^d + v_{q+1,i}^d & \forall q \in B \setminus \{p\}, i \in (I_3^q \cup I_4^q) \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} & (123) \\
e_{qi1}^u + e_{qi2}^u + t_{qi}^d &= l_{q+1,i+1}^d + r_{q+1,i-1}^d & \forall q \in B \setminus \{p\}, i \in (I_3^q \cup I_4^q) \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} & (124) \\
e_{q,\underline{m}_q,1}^u + e_{q,\underline{m}_q,2}^u + t_{q,\underline{m}_q}^d &= l_{q+1,\underline{m}_q+1}^d + v_{q+1,\underline{m}_q}^d & \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_3^q \cup I_4^q \text{ and } \underline{m}_q = \underline{m}_{q+1} & (125) \\
e_{q,\underline{m}_q,1}^u + e_{q,\underline{m}_q,2}^u + t_{q,\underline{m}_q}^d &= l_{q+1,\underline{m}_q+1}^d & \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_3^q \cup I_4^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} & (126) \\
e_{q,\bar{m}_q,1}^u + e_{q,\bar{m}_q,2}^u + t_{q,\bar{m}_q}^d &= r_{q+1,\bar{m}_q-1}^d + v_{q+1,\bar{m}_q}^d & \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_3^q \cup I_4^q \text{ and } \bar{m}_q = \bar{m}_{q+1} & (127) \\
e_{q,\bar{m}_q,1}^u + e_{q,\bar{m}_q,2}^u + t_{q,\bar{m}_q}^d &= r_{q+1,\bar{m}_q-1}^d & \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_3^q \cup I_4^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} & (128) \\
e_{pi1}^u + e_{pi2}^u + t_{pi}^d &= l_{p+1,i+1}^d + r_{p+1,i-1}^d & \forall i \in (I_3^p \cup I_4^p) \setminus \{\underline{m}_p, \bar{m}_p\} & (129) \\
e_{p,\underline{m}_p,1}^u + e_{p,\underline{m}_p,2}^u + t_{p,\underline{m}_p}^d &= l_{p+1,\underline{m}_p+1}^d & \text{if } \underline{m}_p \in I_3^p \cup I_4^p & (130) \\
e_{p,\bar{m}_p,1}^u + e_{p,\bar{m}_p,2}^u + t_{p,\bar{m}_p}^d &= r_{p+1,\bar{m}_p-1}^d & \text{if } \bar{m}_p \in I_3^p \cup I_4^p & (131) \\
e_{qi1}^u + e_{qi3}^u + t_{qi}^d &= l_{q+1,i+1}^d + r_{q+1,i-1}^d + v_{q+1,i}^d & \forall q \in B \setminus \{p\}, i \in (I_0^q \cup I_5^q) \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} & (132)
\end{aligned}$$

$$e_{qi}^u + e_{qi3}^u + t_{qi}^d = l_{q+1,i+1}^d + r_{q+1,i-1}^d \quad \forall q \in B \setminus \{p\}, i \in (I_0^q \cup I_5^q) \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (133)$$

$$e_{q,\underline{m}_q,1}^u + e_{q,\underline{m}_q,3}^u + t_{q,\underline{m}_q}^d = l_{q+1,\underline{m}_q+1}^d + v_{q+1,\underline{m}_q}^d \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_0^q \cup I_5^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \quad (134)$$

$$e_{q,\underline{m}_q,1}^u + e_{q,\underline{m}_q,3}^u + t_{q,\underline{m}_q}^d = l_{q+1,\underline{m}_q+1}^d \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_0^q \cup I_5^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} \quad (135)$$

$$e_{q,\bar{m}_q,1}^u + e_{q,\bar{m}_q,3}^u + t_{q,\bar{m}_q}^d = r_{q+1,\bar{m}_q-1}^d + v_{q+1,\bar{m}_q}^d \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_0^q \cup I_5^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \quad (136)$$

$$e_{q,\bar{m}_q,1}^u + e_{q,\bar{m}_q,3}^u + t_{q,\bar{m}_q}^d = r_{q+1,\bar{m}_q-1}^d \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_0^q \cup I_5^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \quad (137)$$

$$e_{pi}^u + e_{pi3}^u + t_{pi}^d = l_{p+1,i+1}^d + r_{p+1,i-1}^d \quad \forall i \in (I_0^p \cup I_5^p) \setminus \{\underline{m}_p, \bar{m}_p\} \quad (138)$$

$$e_{p,\underline{m}_p,1}^u + e_{p,\underline{m}_p,3}^u + t_{p,\underline{m}_p}^d = l_{p+1,\underline{m}_p+1}^d \quad \text{if } \underline{m}_p \in I_0^p \cup I_5^p \quad (139)$$

$$e_{p,\bar{m}_p,1}^u + e_{p,\bar{m}_p,3}^u + t_{p,\bar{m}_p}^d = r_{p+1,\bar{m}_p-1}^d \quad \text{if } \bar{m}_p \in I_0^p \cup I_5^p \quad (140)$$

- Constraints corresponding to vertices $[u, q, i, 1]$

$$v_{qi}^u + v_{qi}^{u,l} + v_{qi}^{u,r} = e_{qi1}^u + t_{qi}^u \quad \forall q \in B \setminus \{p\}, i \in I_2^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} \quad (141)$$

$$v_{qi}^{u,l} + v_{qi}^{u,r} = e_{qi1}^u + t_{qi}^u \quad \forall q \in B \setminus \{p\}, i \in I_2^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (142)$$

$$v_{q,\underline{m}_q}^u + v_{q,\underline{m}_q}^{u,r} = e_{q,\underline{m}_q,1}^u + t_{q,\underline{m}_q}^u \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_2^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \quad (143)$$

$$v_{q,\underline{m}_q}^{u,r} = e_{q,\underline{m}_q,1}^u + t_{q,\underline{m}_q}^u \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_2^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} \quad (144)$$

$$v_{q,\bar{m}_q}^u + v_{q,\bar{m}_q}^{u,l} = e_{q,\bar{m}_q,1}^u + t_{q,\bar{m}_q}^u \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_2^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \quad (145)$$

$$v_{q,\bar{m}_q}^{u,l} = e_{q,\bar{m}_q,1}^u + t_{q,\bar{m}_q}^u \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_2^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \quad (146)$$

$$v_{pi}^{u,l} + v_{pi}^{u,r} = e_{pi1}^u + t_{pi}^u \quad \forall i \in I_2^p \setminus \{\underline{m}_p, \bar{m}_p\} \quad (147)$$

$$v_{p,\underline{m}_p}^{u,r} = e_{p,\underline{m}_p,1}^u + t_{p,\underline{m}_p}^u \quad \text{if } \underline{m}_p \in I_2^p \quad (148)$$

$$v_{p,\bar{m}_p}^{u,l} = e_{p,\bar{m}_p,1}^u + t_{p,\bar{m}_p}^u \quad \text{if } \bar{m}_p \in I_2^p \quad (149)$$

$$w_{qi1}^u = e_{qi1}^u + t_{qi}^u \quad \forall q \in B, i \in I^q \setminus (I_1^q \cup I_2^q) \quad (150)$$

- Constraints corresponding to vertices $[u, q, i, 2]$

$$v_{qi}^u + v_{qi}^{u,l} + v_{qi}^{u,r} = e_{qi2}^u + w_{qi1}^u \quad \forall q \in B \setminus \{p\}, i \in I_3^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} \quad (151)$$

$$v_{qi}^{u,l} + v_{qi}^{u,r} = e_{qi2}^u + w_{qi1}^u \quad \forall q \in B \setminus \{p\}, i \in I_3^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (152)$$

$$v_{q,\underline{m}_q}^u + v_{q,\underline{m}_q}^{u,r} = e_{q,\underline{m}_q,2}^u + w_{q,\underline{m}_q,1}^u \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_3^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \quad (153)$$

$$v_{q,\underline{m}_q}^{u,r} = e_{q,\underline{m}_q,2}^u + w_{q,\underline{m}_q,1}^u \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_3^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} \quad (154)$$

$$v_{q,\bar{m}_q}^u + v_{q,\bar{m}_q}^{u,l} = e_{q,\bar{m}_q,2}^u + w_{q,\bar{m}_q,1}^u \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_3^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \quad (155)$$

$$v_{q,\bar{m}_q}^{u,l} = e_{q,\bar{m}_q,2}^u + w_{q,\bar{m}_q,1}^u \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_3^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \quad (156)$$

$$v_{pi}^{u,l} + v_{pi}^{u,r} = e_{pi2}^u + w_{pi1}^u \quad \forall i \in I_3^p \setminus \{\underline{m}_p, \bar{m}_p\} \quad (157)$$

$$v_{p,\underline{m}_p}^{u,r} = e_{p,\underline{m}_p,2}^u + w_{p,\underline{m}_p,1}^u \quad \text{if } \underline{m}_p \in I_3^p \quad (158)$$

$$v_{p,\bar{m}_p}^{u,l} = e_{p,\bar{m}_p,2}^u + w_{p,\bar{m}_p,1}^u \quad \text{if } \bar{m}_p \in I_3^p \quad (159)$$

$$w_{qi2}^u = e_{qi2}^u + w_{qi1}^u \quad \forall q \in B, i \in I_4^q \quad (160)$$

$$w_{qi2}^u = w_{qi1}^u \quad \forall q \in B, i \in I_0^q \cup I_5^q \quad (161)$$

- Constraints corresponding to vertices $[u, q, i, 3]$

$$v_{qi}^u + v_{qi}^{u,l} + v_{qi}^{u,r} = w_{qi2}^u \quad \forall q \in B \setminus \{p\}, i \in I_4^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} \quad (162)$$

$$v_{qi}^{u,l} + v_{qi}^{u,r} = w_{qi2}^u \quad \forall q \in B \setminus \{p\}, i \in I_4^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (163)$$

$$v_{q,\underline{m}_q}^u + v_{q,\underline{m}_q}^{u,r} = w_{q,\underline{m}_q,2}^u \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_4^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \quad (164)$$

$$v_{q,\underline{m}_q}^{u,r} = w_{q,\underline{m}_q,2}^u \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_4^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} \quad (165)$$

$$v_{q,\bar{m}_q}^u + v_{q,\bar{m}_q}^{u,l} = w_{q,\bar{m}_q,2}^u \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_4^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \quad (166)$$

$$v_{q,\bar{m}_q}^{u,l} = w_{q,\bar{m}_q,2}^u \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_4^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \quad (167)$$

$$v_{pi}^{u,l} + v_{pi}^{u,r} = w_{pi2}^u \quad \forall i \in I_4^p \setminus \{\underline{m}_p, \bar{m}_p\} \quad (168)$$

$$v_{p,\underline{m}_p}^{u,r} = w_{p,\underline{m}_p,2}^u \quad \text{if } \underline{m}_p \in I_4^p \quad (169)$$

$$v_{p,\bar{m}_p}^{u,l} = w_{p,\bar{m}_p,2}^u \quad \text{if } \bar{m}_p \in I_4^p \quad (170)$$

$$v_{qi}^u + v_{qi}^{u,l} + v_{qi}^{u,r} = e_{qi3}^u + w_{qi2}^u \quad \forall q \in B \setminus \{p\}, i \in I_5^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} \quad (171)$$

$$v_{qi}^{u,l} + v_{qi}^{u,r} = e_{qi3}^u + w_{qi2}^u \quad \forall q \in B \setminus \{p\}, i \in I_5^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (172)$$

$$v_{q,\underline{m}_q}^u + v_{q,\underline{m}_q}^{u,r} = e_{q,\underline{m}_q,3}^u + w_{q,\underline{m}_q,2}^u \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_5^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \quad (173)$$

$$v_{q,\underline{m}_q}^{u,r} = e_{q,\underline{m}_q,3}^u + w_{q,\underline{m}_q,2}^u \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_5^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} \quad (174)$$

$$v_{q,\bar{m}_q}^u + v_{q,\bar{m}_q}^{u,l} = e_{q,\bar{m}_q,3}^u + w_{q,\bar{m}_q,2}^u \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_5^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \quad (175)$$

$$v_{q,\bar{m}_q}^{u,l} = e_{q,\bar{m}_q,3}^u + w_{q,\bar{m}_q,2}^u \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_5^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \quad (176)$$

$$v_{pi}^{u,l} + v_{pi}^{u,r} = e_{pi3}^u + w_{pi2}^u \quad \forall i \in I_5^p \setminus \{\underline{m}_p, \bar{m}_p\} \quad (177)$$

$$v_{p,\underline{m}_p}^{u,r} = e_{p,\underline{m}_p,3}^u + w_{p,\underline{m}_p,2}^u \quad \text{if } \underline{m}_p \in I_5^p \quad (178)$$

$$v_{p,\bar{m}_p}^{u,l} = e_{p,\bar{m}_p,3}^u + w_{p,\bar{m}_p,2}^u \quad \text{if } \bar{m}_p \in I_5^p \quad (179)$$

$$w_{qi3}^u = e_{qi3}^u + w_{qi2}^u \quad \forall q \in B, i \in I_0^q \quad (180)$$

- Constraints corresponding to vertices $[u, q, i, 4]$

$$v_{qi}^u + v_{qi}^{u,l} + v_{qi}^{u,r} = w_{qi3}^u \quad \forall q \in B \setminus \{p\}, i \in I_0^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} \quad (181)$$

$$v_{qi}^{u,l} + v_{qi}^{u,r} = w_{qi3}^u \quad \forall q \in B \setminus \{p\}, i \in I_0^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (182)$$

$$v_{q,\underline{m}_q}^u + v_{q,\underline{m}_q}^{u,r} = w_{q,\underline{m}_q,3}^u \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_0^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \quad (183)$$

$$v_{q,\underline{m}_q}^{u,r} = w_{q,\underline{m}_q,3}^u \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_0^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} \quad (184)$$

$$v_{q,\bar{m}_q}^u + v_{q,\bar{m}_q}^{u,l} = w_{q,\bar{m}_q,3}^u \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_0^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \quad (185)$$

$$v_{q,\bar{m}_q}^{u,l} = w_{q,\bar{m}_q,3}^u \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_0^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \quad (186)$$

$$v_{pi}^{u,l} + v_{pi}^{u,r} = w_{pi3}^u \quad \forall i \in I_0^p \setminus \{\underline{m}_p, \bar{m}_p\} \quad (187)$$

$$v_{p,\underline{m}_p}^{u,r} = w_{p,\underline{m}_p,3}^u \quad \text{if } \underline{m}_p \in I_0^p \quad (188)$$

$$v_{p,\bar{m}_p}^{u,l} = w_{p,\bar{m}_p,3}^u \quad \text{if } \bar{m}_p \in I_0^p \quad (189)$$

- Constraints corresponding to vertices $[d, q, i, 1]$

$$v_{1i}^{d,l} + v_{1i}^{d,r} = e_{1i1}^d + t_{1i}^d \quad \forall i \in I_2^1 \setminus \{\bar{m}_1\} \quad (190)$$

$$v_{1,\bar{m}_1}^{d,l} = e_{1,\bar{m}_1,1}^d + t_{1,\bar{m}_1}^d \quad \text{if } \bar{m}_1 \in I_2^1 \quad (191)$$

$$v_{qi}^d + v_{qi}^{d,l} + v_{qi}^{d,r} = e_{qi1}^d + t_{qi}^d \quad \forall q \in B \setminus \{1\}, i \in I_2^q \setminus \{\underline{m}_q, \bar{m}_q\} \quad (192)$$

$$v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,l} + v_{q,\underline{m}_q}^{d,r} = e_{q,\underline{m}_q,1}^d + t_{q,\underline{m}_q}^d \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_2^q \text{ and } \underline{m}_{q-1} < \underline{m}_q \quad (193)$$

$$v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,r} = e_{q,\underline{m}_q,1}^d + t_{q,\underline{m}_q}^d \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_2^q \text{ and } \underline{m}_{q-1} = \underline{m}_q \quad (194)$$

$$v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} + v_{q,\bar{m}_q}^{d,r} = e_{q,\bar{m}_q,1}^d + t_{q,\bar{m}_q}^d \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_2^q \text{ and } \bar{m}_{q-1} > \bar{m}_q \quad (195)$$

$$v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} = e_{q,\bar{m}_q,1}^d + t_{q,\bar{m}_q}^d \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_2^q \text{ and } \bar{m}_{q-1} = \bar{m}_q \quad (196)$$

$$w_{qi1}^d = e_{qi1}^d + t_{qi}^d \quad \forall q \in B, i \in I^q \setminus (I_1^q \cup I_2^q) \quad (197)$$

- Constraints corresponding to vertices $[d, q, i, 2]$

$$v_{1i}^{d,l} + v_{1i}^{d,r} = e_{1i2}^d + w_{1i1}^d \quad \forall i \in I_3^1 \setminus \{\bar{m}_1\} \quad (198)$$

$$v_{1,\bar{m}_1}^{d,l} = e_{1,\bar{m}_1,2}^d + w_{1,\bar{m}_1,1}^d \quad \text{if } \bar{m}_1 \in I_3^1 \quad (199)$$

$$v_{qi}^d + v_{qi}^{d,l} + v_{qi}^{d,r} = e_{qi2}^d + w_{qi1}^d \quad \forall q \in B \setminus \{1\}, i \in I_3^q \setminus \{\underline{m}_q, \bar{m}_q\} \quad (200)$$

$$v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,l} + v_{q,\underline{m}_q}^{d,r} = e_{q,\underline{m}_q,2}^d + w_{q,\underline{m}_q,1}^d \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_3^q \text{ and } \underline{m}_{q-1} < \underline{m}_q \quad (201)$$

$$v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,r} = e_{q,\underline{m}_q,2}^d + w_{q,\underline{m}_q,1}^d \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_3^q \text{ and } \underline{m}_{q-1} = \underline{m}_q \quad (202)$$

$$v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} + v_{q,\bar{m}_q}^{d,r} = e_{q,\bar{m}_q,2}^d + w_{q,\bar{m}_q,1}^d \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_3^q \text{ and } \bar{m}_{q-1} > \bar{m}_q \quad (203)$$

$$v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} = e_{q,\bar{m}_q,2}^d + w_{q,\bar{m}_q,1}^d \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_3^q \text{ and } \bar{m}_{q-1} = \bar{m}_q \quad (204)$$

$$w_{qi2}^d = w_{qi1}^d \quad \forall q \in B, i \in I_0^q \cup I_4^q \quad (205)$$

$$w_{qi2}^d = e_{qi2}^d + w_{qi1}^d \quad \forall q \in B, i \in I_5^q \quad (206)$$

- Constraints corresponding to vertices $[d, q, i, 3]$

$$v_{1i}^{d,l} + v_{1i}^{d,r} = e_{1i3}^d + w_{1i2}^d \quad \forall i \in I_4^1 \setminus \{\bar{m}_1\} \quad (207)$$

$$v_{1,\bar{m}_1}^{d,l} = e_{1,\bar{m}_1,3}^d + w_{1,\bar{m}_1,2}^d \quad \text{if } \bar{m}_1 \in I_4^1 \quad (208)$$

$$v_{qi}^d + v_{qi}^{d,l} + v_{qi}^{d,r} = e_{qi3}^d + w_{qi2}^d \quad \forall q \in B \setminus \{1\}, i \in I_4^q \setminus \{\underline{m}_q, \bar{m}_q\} \quad (209)$$

$$v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,l} + v_{q,\underline{m}_q}^{d,r} = e_{q,\underline{m}_q,3}^d + w_{q,\underline{m}_q,2}^d \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_4^q \text{ and } \underline{m}_{q-1} < \underline{m}_q \quad (210)$$

$$v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,r} = e_{q,\underline{m}_q,3}^d + w_{q,\underline{m}_q,2}^d \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_4^q \text{ and } \underline{m}_{q-1} = \underline{m}_q \quad (211)$$

$$v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} + v_{q,\bar{m}_q}^{d,r} = e_{q,\bar{m}_q,3}^d + w_{q,\bar{m}_q,2}^d \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_4^q \text{ and } \bar{m}_{q-1} > \bar{m}_q \quad (212)$$

$$v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} = e_{q,\bar{m}_q,3}^d + w_{q,\bar{m}_q,2}^d \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_4^q \text{ and } \bar{m}_{q-1} = \bar{m}_q \quad (213)$$

$$v_{1i}^{d,l} + v_{1i}^{d,r} = w_{1i2}^d \quad \forall i \in I_5^1 \setminus \{\bar{m}_1\} \quad (214)$$

$$v_{1,\bar{m}_1}^{d,l} = w_{1,\bar{m}_1,2}^d \quad \text{if } \bar{m}_1 \in I_5^1 \quad (215)$$

$$v_{qi}^d + v_{qi}^{d,l} + v_{qi}^{d,r} = w_{qi2}^d \quad \forall q \in B \setminus \{1\}, i \in I_5^q \setminus \{\underline{m}_q, \bar{m}_q\} \quad (216)$$

$$v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,l} + v_{q,\underline{m}_q}^{d,r} = w_{q,\underline{m}_q,2}^d \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_5^q \text{ and } \underline{m}_{q-1} < \underline{m}_q \quad (217)$$

$$v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,r} = w_{q,\underline{m}_q,2}^d \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_5^q \text{ and } \underline{m}_{q-1} = \underline{m}_q \quad (218)$$

$$v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} + v_{q,\bar{m}_q}^{d,r} = w_{q,\bar{m}_q,2}^d \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_5^q \text{ and } \bar{m}_{q-1} > \bar{m}_q \quad (219)$$

$$v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} = w_{q,\bar{m}_q}^d \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_5^q \text{ and } \bar{m}_{q-1} = \bar{m}_q \quad (220)$$

$$w_{qi3}^d = e_{qi3}^d + w_{qi2}^d \quad \forall q \in B, i \in I_0^q \quad (221)$$

- Constraints corresponding to vertices $[d, q, i, 4]$

$$v_{1i}^{d,l} + v_{1i}^{d,r} = w_{1i3}^d \quad \forall i \in I_0^1 \setminus \{\bar{m}_1\} \quad (222)$$

$$v_{1,\bar{m}_1}^{d,l} = w_{1,\bar{m}_1,3}^d \quad \text{if } \bar{m}_1 \in I_0^1 \quad (223)$$

$$v_{qi}^d + v_{qi}^{d,l} + v_{qi}^{d,r} = w_{qi3}^d \quad \forall q \in B \setminus \{1\}, i \in I_0^q \setminus \{\underline{m}_q, \bar{m}_q\} \quad (224)$$

$$v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,l} + v_{q,\underline{m}_q}^{d,r} = w_{q,\underline{m}_q,3}^d \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_0^q \text{ and } \underline{m}_{q-1} < \underline{m}_q \quad (225)$$

$$v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,r} = w_{q,\underline{m}_q,3}^d \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_0^q \text{ and } \underline{m}_{q-1} = \underline{m}_q \quad (226)$$

$$v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} + v_{q,\bar{m}_q}^{d,r} = w_{q,\bar{m}_q,3}^d \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_0^q \text{ and } \bar{m}_{q-1} > \bar{m}_q \quad (227)$$

$$v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} = w_{q,\bar{m}_q,3}^d \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_0^q \text{ and } \bar{m}_{q-1} = \bar{m}_q \quad (228)$$

Subtour Elimination Constraints:

- Constraints corresponding to vertices $[r, q, i]$

$$\tilde{r}_{1,i-1}^r + \tilde{v}_{1i}^{d,r} - (\tilde{r}_{1i}^r + \tilde{r}_{1i}^u) = r_{1i}^r + r_{1i}^u \quad \forall i \in I^1 \setminus \{\underline{m}_1, \bar{m}_1 - 1, \bar{m}_1\} \quad (229)$$

$$\tilde{y}_r^0 + \tilde{v}_{1,\underline{m}_1}^{d,r} - (\tilde{r}_{1,\underline{m}_1}^r + \tilde{r}_{1,\underline{m}_1}^u) = r_{1,\underline{m}_1}^r + r_{1,\underline{m}_1}^u \quad (230)$$

$$\tilde{r}_{1,\bar{m}_1-1}^r + \tilde{v}_{1,\bar{m}_1-1}^{d,r} - \tilde{r}_{1,\bar{m}_1-1}^u = r_{1,\bar{m}_1-1}^u \quad (231)$$

$$\tilde{r}_{q,i-1}^r + \tilde{v}_{q-1,i}^{u,r} + \tilde{v}_{qi}^{d,r} - (\tilde{r}_{qi}^r + \tilde{r}_{qi}^u + \tilde{r}_{qi}^d) = r_{qi}^r + r_{qi}^u + r_{qi}^d \quad \forall q \in F \setminus \{1, p+1\}, i \in I^q \setminus \{\underline{m}_{q-1}, \bar{m}_{q-1} - 1, \bar{m}_{q-1}\} \text{ with } \underline{m}_q - 1 < i < \bar{m}_q \quad (232)$$

$$\tilde{r}_{q,i-1}^r + \tilde{v}_{q-1,i}^{u,r} - (\tilde{r}_{qi}^r + \tilde{r}_{qi}^d) = r_{qi}^r + r_{qi}^d \quad \forall q \in F \setminus \{1, p+1\}, i \in I^q \setminus \{\underline{m}_{q-1}, \bar{m}_{q-1} - 1, \bar{m}_{q-1}\} \text{ with } i < \underline{m}_q - 1 \text{ or } i > \bar{m}_q \quad (233)$$

$$\tilde{r}_{q,i-1}^r + \tilde{v}_{q-1,i}^{u,r} + \tilde{v}_{qi}^{d,r} - (\tilde{r}_{qi}^r + \tilde{r}_{qi}^d) = r_{qi}^r + r_{qi}^d \quad \forall q \in F \setminus \{1, p+1\}, i \in I^q \setminus \{\underline{m}_{q-1}, \bar{m}_{q-1} - 1, \bar{m}_{q-1}\} \text{ with } i = \bar{m}_q \quad (234)$$

$$\tilde{r}_{q,i-1}^r + \tilde{v}_{q-1,i}^{u,r} - (\tilde{r}_{qi}^r + \tilde{r}_{qi}^u + \tilde{r}_{qi}^d) = r_{qi}^r + r_{qi}^u + r_{qi}^d \quad \forall q \in F \setminus \{1, p+1\}, i \in I^q \setminus \{\underline{m}_{q-1}, \bar{m}_{q-1} - 1, \bar{m}_{q-1}\} \text{ with } i = \underline{m}_q - 1 \quad (235)$$

$$\tilde{v}_{q-1,\underline{m}_{q-1}}^{u,r} + \tilde{v}_{q,\underline{m}_{q-1}}^{d,r} - (\tilde{r}_{q,\underline{m}_{q-1}}^r + \tilde{r}_{q,\underline{m}_{q-1}}^u + \tilde{r}_{q,\underline{m}_{q-1}}^d) = r_{q,\underline{m}_{q-1}}^r + r_{q,\underline{m}_{q-1}}^u + r_{q,\underline{m}_{q-1}}^d \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \underline{m}_q = \underline{m}_{q-1} \quad (236)$$

$$\tilde{v}_{q-1,\underline{m}_{q-1}}^{u,r} - (\tilde{r}_{q,\underline{m}_{q-1}}^r + \tilde{r}_{q,\underline{m}_{q-1}}^u + \tilde{r}_{q,\underline{m}_{q-1}}^d) = r_{q,\underline{m}_{q-1}}^r + r_{q,\underline{m}_{q-1}}^u + r_{q,\underline{m}_{q-1}}^d \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \underline{m}_q = \underline{m}_{q-1} + 1 \quad (237)$$

$$\tilde{v}_{q-1,\underline{m}_{q-1}}^{u,r} - (\tilde{r}_{q,\underline{m}_{q-1}}^r + \tilde{r}_{q,\underline{m}_{q-1}}^d) = r_{q,\underline{m}_{q-1}}^r + r_{q,\underline{m}_{q-1}}^d \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \underline{m}_q > \underline{m}_{q-1} + 1 \quad (238)$$

$$\tilde{r}_{q,\bar{m}_{q-1}-2}^r + \tilde{v}_{q-1,\bar{m}_{q-1}-1}^{u,r} + \tilde{v}_{q,\bar{m}_{q-1}-1}^{d,r} - (\tilde{r}_{q,\bar{m}_{q-1}-1}^u + \tilde{r}_{q,\bar{m}_{q-1}-1}^d) = r_{q,\bar{m}_{q-1}-1}^u + r_{q,\bar{m}_{q-1}-1}^d \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \bar{m}_q = \bar{m}_{q-1} \quad (239)$$

$$\tilde{r}_{q,\bar{m}_{q-1}-2}^r + \tilde{v}_{q-1,\bar{m}_{q-1}-1}^{u,r} + \tilde{v}_{q,\bar{m}_{q-1}-1}^{d,r} - \tilde{r}_{q,\bar{m}_{q-1}-1}^d = r_{q,\bar{m}_{q-1}-1}^d \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \bar{m}_q = \bar{m}_{q-1} - 1 \quad (240)$$

$$\tilde{r}_{q,\bar{m}_{q-1}-2}^r + \tilde{v}_{q-1,\bar{m}_{q-1}-1}^{u,r} - \tilde{r}_{q,\bar{m}_{q-1}-1}^d = r_{q,\bar{m}_{q-1}-1}^d \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \bar{m}_q < \bar{m}_{q-1} - 1 \quad (241)$$

$$\tilde{r}_{p+1,i-1}^r + \tilde{v}_{pi}^{u,r} - (\tilde{r}_{p+1,i}^r + \tilde{r}_{p+1,i}^d) = r_{p+1,i}^r + r_{p+1,i}^d \quad \forall i \in I^p \setminus \{\underline{m}_p, \bar{m}_p - 1, \bar{m}_p\} \quad (242)$$

$$\tilde{v}_{p,\underline{m}_p}^{u,r} - (\tilde{r}_{p+1,\underline{m}_p}^r + \tilde{r}_{p+1,\underline{m}_p}^d) = r_{p+1,\underline{m}_p}^r + r_{p+1,\underline{m}_p}^d \quad (243)$$

$$\tilde{r}_{p+1,\bar{m}_p-2}^r + \tilde{v}_{p,\bar{m}_p-1}^{u,r} - \tilde{r}_{p+1,\bar{m}_p-1}^d = r_{p+1,\bar{m}_p-1}^d \quad (244)$$

- Constraints corresponding to vertices $[l, q, i]$

$$\tilde{l}_{1,i+1}^l + \tilde{v}_{1i}^{d,l} - (\tilde{l}_{1i}^l + \tilde{l}_{1i}^u) = l_{1i}^l + l_{1i}^u \quad \forall i \in I^l \setminus \{\underline{m}_1, \bar{m}_1\} \quad (245)$$

$$\tilde{y}_l^0 + \tilde{l}_{1,\underline{m}_1+1}^l + \tilde{v}_{1,\underline{m}_1}^{d,l} - \tilde{y}_0^l = y_0^l \quad (246)$$

$$\tilde{v}_{1,\bar{m}_1}^{d,l} - (\tilde{l}_{1,\bar{m}_1}^l + \tilde{l}_{1,\bar{m}_1}^u) = l_{1,\bar{m}_1}^l + l_{1,\bar{m}_1}^u \quad (247)$$

$$\tilde{l}_{q,i+1}^l + \tilde{v}_{q,i}^{d,l} + \tilde{v}_{q-1,i}^{u,l} - (\tilde{l}_{qi}^l + \tilde{l}_{qi}^u + \tilde{l}_{qi}^d) = l_{qi}^l + l_{qi}^u + l_{qi}^d \quad \forall q \in F \setminus \{1, p+1\}, i \in I^q \setminus \{\underline{m}_{q-1}, \underline{m}_{q-1} + 1, \bar{m}_{q-1}\} \text{ with } \underline{m}_q < i < \bar{m}_q + 1 \quad (248)$$

$$\tilde{l}_{q,i+1}^l + \tilde{v}_{q-1,i}^{u,l} - (\tilde{l}_{qi}^l + \tilde{l}_{qi}^d) = l_{qi}^l + l_{qi}^d \quad \forall q \in F \setminus \{1, p+1\}, i \in I^q \setminus \{\underline{m}_{q-1}, \underline{m}_{q-1} + 1, \bar{m}_{q-1}\} \text{ with } i < \underline{m}_q \text{ or } i > \bar{m}_q + 1 \quad (249)$$

$$\tilde{l}_{q,i+1}^l + \tilde{v}_{q,i}^{d,l} + \tilde{v}_{q-1,i}^{u,l} - (\tilde{l}_{qi}^l + \tilde{l}_{qi}^d) = l_{qi}^l + l_{qi}^d \quad \forall q \in F \setminus \{1, p+1\}, i \in I^q \setminus \{\underline{m}_{q-1}, \underline{m}_{q-1} + 1, \bar{m}_{q-1}\} \text{ with } i = \underline{m}_q \quad (250)$$

$$\tilde{l}_{q,i+1}^l + \tilde{v}_{q-1,i}^{u,l} - (\tilde{l}_{qi}^l + \tilde{l}_{qi}^u + \tilde{l}_{qi}^d) = l_{qi}^l + l_{qi}^u + l_{qi}^d \quad \forall q \in F \setminus \{1, p+1\}, i \in I^q \setminus \{\underline{m}_{q-1}, \underline{m}_{q-1} + 1, \bar{m}_{q-1}\} \text{ with } i = \bar{m}_q + 1 \quad (251)$$

$$\tilde{l}_{q,\underline{m}_{q-1}+2}^l + \tilde{v}_{q,\underline{m}_{q-1}+1}^{d,l} + \tilde{v}_{q-1,\underline{m}_{q-1}+1}^{u,l} - (\tilde{l}_{q,\underline{m}_{q-1}+1}^l + \tilde{l}_{q,\underline{m}_{q-1}+1}^d) = l_{q,\underline{m}_{q-1}+1}^u + l_{q,\underline{m}_{q-1}+1}^d \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \underline{m}_q = \underline{m}_{q-1} \quad (252)$$

$$\tilde{l}_{q,\underline{m}_{q-1}+2}^l + \tilde{v}_{q,\underline{m}_{q-1}+1}^{d,l} + \tilde{v}_{q-1,\underline{m}_{q-1}+1}^{u,l} - \tilde{l}_{q,\underline{m}_{q-1}+1}^d = l_{q,\underline{m}_{q-1}+1}^d \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \underline{m}_q = \underline{m}_{q-1} + 1 \quad (253)$$

$$\tilde{l}_{q,\underline{m}_{q-1}+2}^l + \tilde{v}_{q-1,\underline{m}_{q-1}+1}^{u,l} - \tilde{l}_{q,\underline{m}_{q-1}+1}^d = l_{q,\underline{m}_{q-1}+1}^d \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \underline{m}_q > \underline{m}_{q-1} + 1 \quad (254)$$

$$\tilde{v}_{q,\bar{m}_{q-1}}^{d,l} + \tilde{v}_{q-1,\bar{m}_{q-1}}^{u,l} - (\tilde{l}_{q,\bar{m}_{q-1}}^l + \tilde{l}_{q,\bar{m}_{q-1}}^u + \tilde{l}_{q,\bar{m}_{q-1}}^d) = l_{q,\bar{m}_{q-1}}^l + l_{q,\bar{m}_{q-1}}^u + l_{q,\bar{m}_{q-1}}^d \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \bar{m}_q = \bar{m}_{q-1} \quad (255)$$

$$\tilde{v}_{q-1,\bar{m}_{q-1}}^{u,l} - (\tilde{l}_{q,\bar{m}_{q-1}}^l + \tilde{l}_{q,\bar{m}_{q-1}}^u + \tilde{l}_{q,\bar{m}_{q-1}}^d) = l_{q,\bar{m}_{q-1}}^l + l_{q,\bar{m}_{q-1}}^u + l_{q,\bar{m}_{q-1}}^d \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \bar{m}_q = \bar{m}_{q-1} - 1 \quad (256)$$

$$\tilde{v}_{q-1,\bar{m}_{q-1}}^{u,l} - (\tilde{l}_{q,\bar{m}_{q-1}}^l + \tilde{l}_{q,\bar{m}_{q-1}}^d) = l_{q,\bar{m}_{q-1}}^l + l_{q,\bar{m}_{q-1}}^d \quad \forall q \in F \setminus \{1, p+1\} \text{ with } \bar{m}_q < \bar{m}_{q-1} - 1 \quad (257)$$

$$\tilde{l}_{p+1,i+1}^l + \tilde{v}_{pi}^{u,l} - (\tilde{l}_{p+1,i}^l + \tilde{l}_{p+1,i}^d) = l_{p+1,i}^l + l_{p+1,i}^d \quad \forall i \in I^p \setminus \{\underline{m}_p, \underline{m}_p + 1, \bar{m}_p\} \quad (258)$$

$$\tilde{l}_{p+1,\underline{m}_p+2}^l + \tilde{v}_{p,\underline{m}_p+1}^{u,l} - \tilde{l}_{p+1,\underline{m}_p+1}^d = l_{p+1,\underline{m}_p+1}^d \quad (259)$$

$$\tilde{v}_{p,\bar{m}_p}^{u,l} - (\tilde{l}_{p+1,\bar{m}_p}^l + \tilde{l}_{p+1,\bar{m}_p}^d) = l_{p+1,\bar{m}_p}^l + l_{p+1,\bar{m}_p}^d \quad (260)$$

- Constraints corresponding to vertices $[u, q, i]$

$$\tilde{l}_{q,i+1}^u + \tilde{r}_{q,i-1}^u + \tilde{v}_{q-1,i}^{u,l} - (\tilde{v}_{qi}^u + \tilde{v}_{qi}^{u,l} + \tilde{v}_{qi}^{u,r}) = v_{qi}^u + v_{qi}^{u,l} + v_{qi}^{u,r} \quad (261)$$

$$\forall q \in B \setminus \{1, p\}, i \in I_1^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i \geq \underline{m}_{q+1}, i \leq \bar{m}_{q+1} \quad (262)$$

$$\tilde{l}_{1,\underline{m}_1+1}^u + \tilde{y}_u^0 - (\tilde{v}_{1,\underline{m}_1}^u + \tilde{v}_{1,\underline{m}_1}^{u,r}) = v_{1,\underline{m}_1}^u + v_{1,\underline{m}_1}^{u,r} \quad \text{if } \underline{m}_1 = \underline{m}_2 \text{ and } \underline{m}_1 \in I_1^l \quad (263)$$

$$\tilde{r}_{q,\bar{m}_q-1}^u + \tilde{v}_{q-1,\bar{m}_q}^u - (\tilde{v}_{q,\bar{m}_q}^u + \tilde{v}_{q,\bar{m}_q}^{u,l}) = v_{q,\bar{m}_q}^u + v_{q,\bar{m}_q}^{u,l} \quad \forall q \in B \setminus \{1, p\} \text{ with } \bar{m}_q \in I_1^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \text{ and } \bar{m}_q = \bar{m}_{q-1} \quad (264)$$

$$\tilde{r}_{q,\bar{m}_q-1}^u + \tilde{l}_{q,\bar{m}_q+1}^u + \tilde{v}_{q-1,\bar{m}_q}^u - (\tilde{v}_{q,\bar{m}_q}^u + \tilde{v}_{q,\bar{m}_q}^{u,l}) = v_{q,\bar{m}_q}^u + v_{q,\bar{m}_q}^{u,l} \quad \forall q \in B \setminus \{1, p\} \text{ with } \bar{m}_q \in I_1^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \text{ and } \bar{m}_q \neq \bar{m}_{q-1} \quad (265)$$

$$\tilde{l}_{q,i+1}^u + \tilde{r}_{q,i-1}^u + \tilde{v}_{q-1,i}^u - (\tilde{v}_{qi}^{u,l} + \tilde{v}_{qi}^{u,r}) = v_{q,i}^{u,l} + v_{q,i}^{u,r} \quad \forall q \in B \setminus \{1, p\}, i \in I_1^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (266)$$

$$\tilde{l}_{1,\underline{m}_1+1}^u + \tilde{y}_u^0 - \tilde{v}_{1,\underline{m}_1}^{u,r} = v_{1,\underline{m}_1}^{u,r} \quad \text{if } \underline{m}_1 \neq \underline{m}_2 \text{ and } \underline{m}_1 \in I_1^1 \quad (267)$$

$$\tilde{r}_{q,\bar{m}_q-1}^u + \tilde{v}_{q-1,\bar{m}_q}^u - \tilde{v}_{q,\bar{m}_q}^{u,l} = v_{q,\bar{m}_q}^{u,l} \quad \forall q \in B \setminus \{1, p\} \text{ with } \bar{m}_q \in I_1^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \text{ and } \bar{m}_q = \bar{m}_{q-1} \quad (268)$$

$$\tilde{r}_{q,\bar{m}_q-1}^u + \tilde{l}_{q,\bar{m}_q+1}^u + \tilde{v}_{q-1,\bar{m}_q}^u - \tilde{v}_{q,\bar{m}_q}^{u,l} = v_{q,\bar{m}_q}^{u,l} \quad \forall q \in B \setminus \{1, p\} \text{ with } \bar{m}_q \in I_1^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \text{ and } \bar{m}_q \neq \bar{m}_{q-1} \quad (269)$$

$$\tilde{l}_{q,\underline{m}_q+1}^u + \tilde{r}_{q-1,\underline{m}_q}^u - (\tilde{v}_{q,\underline{m}_q}^u + \tilde{v}_{q,\underline{m}_q}^{u,r}) = v_{q,\underline{m}_q}^u + v_{q,\underline{m}_q}^{u,r} \quad (270)$$

$$\forall q \in B \setminus \{1, p\} \text{ with } \underline{m}_q \in I_1^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \text{ and } \underline{m}_q = \underline{m}_{q-1} \quad (271)$$

$$\tilde{r}_{q,\underline{m}_q-1}^u + \tilde{l}_{q,\underline{m}_q+1}^u + \tilde{v}_{q-1,\underline{m}_q}^u - (\tilde{v}_{q,\underline{m}_q}^u + \tilde{v}_{q,\underline{m}_q}^{u,r}) = v_{q,\underline{m}_q}^u + v_{q,\underline{m}_q}^{u,r} \quad (272)$$

$$\forall q \in B \setminus \{1, p\} \text{ with } \underline{m}_q \in I_1^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \text{ and } \underline{m}_q \neq \underline{m}_{q-1} \quad (273)$$

$$\tilde{l}_{q,\underline{m}_q+1}^u + \tilde{r}_{q-1,\underline{m}_q}^u - \tilde{v}_{q,\underline{m}_q}^{u,r} = v_{q,\underline{m}_q}^{u,r} \quad \forall q \in B \setminus \{1, p\} \text{ with } \underline{m}_q \neq \underline{m}_{q+1} \text{ and } \underline{m}_q \in I_1^q \quad (274)$$

$$\tilde{l}_{p,i+1}^u + \tilde{r}_{p,i-1}^u + \tilde{v}_{p-1,i}^u - (\tilde{v}_{pi}^{u,l} + \tilde{v}_{pi}^{u,r}) = v_{pi}^{u,l} + v_{pi}^{u,r} \quad \forall i \in I_1^p \setminus \{\underline{m}_p, \bar{m}_p\} \quad (275)$$

$$\tilde{l}_{p,\underline{m}_p+1}^u + \tilde{r}_{p-1,\underline{m}_p}^u - \tilde{v}_{p,\underline{m}_p}^{u,r} = v_{p,\underline{m}_p}^{u,r} \quad \text{if } \underline{m}_p \in I_1^p \quad (276)$$

$$\tilde{r}_{p,\bar{m}_p-1}^u + \tilde{v}_{p-1,\bar{m}_p}^u - \tilde{v}_{p,\bar{m}_p}^{u,l} = v_{p,\bar{m}_p}^{u,l} \quad \text{if } \bar{m}_p \in I_1^p \quad (277)$$

$$\tilde{l}_{1,i+1}^u + \tilde{r}_{1,i-1}^u - (\tilde{v}_{1i}^u + \tilde{v}_{1i}^{u,l} + \tilde{v}_{1i}^{u,r}) = v_{1i}^u + v_{1i}^{u,l} + v_{1i}^{u,r} \quad \forall i \in I_1^1 \setminus \{\underline{m}_1, \bar{m}_1\} \text{ with } \underline{m}_2 \leq i \leq \bar{m}_2 \quad (278)$$

$$\tilde{l}_{1,i+1}^u + \tilde{r}_{1,i-1}^u - (\tilde{v}_{1i}^{u,l} + \tilde{v}_{1i}^{u,r}) = v_{1i}^{u,l} + v_{1i}^{u,r} \quad \forall i \in I_1^1 \setminus \{\underline{m}_1, \bar{m}_1\} \text{ with } i < \underline{m}_2 \text{ or } i > \bar{m}_2 \quad (279)$$

$$\tilde{r}_{1,\bar{m}_1-1}^u - (\tilde{v}_{1,\bar{m}_1}^u + \tilde{v}_{1,\bar{m}_1}^{u,l}) = v_{1,\bar{m}_1}^u + v_{1,\bar{m}_1}^{u,l} \quad \text{if } \bar{m}_1 = \bar{m}_2 \text{ and } \bar{m}_1 \in I_1^1 \quad (280)$$

$$\tilde{r}_{1,\bar{m}_1-1}^u - \tilde{v}_{1,\bar{m}_1}^{u,l} = v_{1,\bar{m}_1}^{u,l} \quad \text{if } \bar{m}_1 \neq \bar{m}_2 \text{ and } \bar{m}_1 \in I_1^1 \quad (281)$$

$$\tilde{l}_{1,i+1}^u + \tilde{r}_{1,i-1}^u - (\tilde{t}_{1i}^u + \tilde{e}_{1i}^d) = t_{1i}^u + e_{1i}^d \quad \forall i \in I_2^1 \setminus \{\underline{m}_1, \bar{m}_1\} \quad (282)$$

$$\tilde{l}_{1,\underline{m}_1+1}^u + \tilde{y}_u^0 - (\tilde{t}_{1,\underline{m}_1}^u + \tilde{e}_{1,\underline{m}_1,1}^d) = t_{1,\underline{m}_1}^u + e_{1,\underline{m}_1,1}^d \quad \text{if } \underline{m}_1 \in I_2^1 \quad (283)$$

$$\tilde{r}_{1,\bar{m}_1-1}^u - (\tilde{t}_{1,\bar{m}_1}^u + \tilde{e}_{1,\bar{m}_1,1}^d) = t_{1,\bar{m}_1}^u + e_{1,\bar{m}_1,1}^d \quad \text{if } \bar{m}_1 \in I_2^1 \quad (284)$$

$$\tilde{l}_{q,i+1}^u + \tilde{r}_{q,i-1}^u + \tilde{v}_{q-1,i}^u - (\tilde{t}_{qi}^u + \tilde{e}_{qi}^d) = t_{qi}^u + e_{qi}^d \quad \forall q \in B \setminus \{1\}, i \in I_2^q \setminus \{\underline{m}_q, \bar{m}_q\} \quad (285)$$

$$\tilde{l}_{q,\underline{m}_q+1}^u + \tilde{r}_{q-1,\underline{m}_q}^u - (\tilde{t}_{q,\underline{m}_q}^u + \tilde{e}_{q,\underline{m}_q,1}^d) = t_{q,\underline{m}_q}^u + e_{q,\underline{m}_q,1}^d \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_2^q \text{ and } \underline{m}_q = \underline{m}_{q-1} \quad (286)$$

$$\tilde{r}_{q,\underline{m}_q-1}^u + \tilde{l}_{q,\underline{m}_q+1}^u + \tilde{v}_{q-1,\underline{m}_q}^u - (\tilde{t}_{q,\underline{m}_q}^u + \tilde{e}_{q,\underline{m}_q,1}^d) = t_{q,\underline{m}_q}^u + e_{q,\underline{m}_q,1}^d \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_2^q \text{ and } \underline{m}_q \neq \underline{m}_{q-1} \quad (287)$$

$$\tilde{r}_{q,\bar{m}_q-1}^u + \tilde{v}_{q-1,\bar{m}_q}^u - (\tilde{t}_{q,\bar{m}_q}^u + \tilde{e}_{q,\bar{m}_q,1}^d) = t_{q,\bar{m}_q}^u + e_{q,\bar{m}_q,1}^d \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_2^q \text{ and } \bar{m}_q = \bar{m}_{q-1} \quad (288)$$

$$\tilde{r}_{q,\bar{m}_q-1}^u + \tilde{l}_{q,\bar{m}_q+1}^u + \tilde{v}_{q-1,\bar{m}_q}^u - (\tilde{t}_{q,\bar{m}_q}^u + \tilde{e}_{q,\bar{m}_q,1}^d) = t_{q,\bar{m}_q}^u + e_{q,\bar{m}_q,1}^d \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_2^q \text{ and } \bar{m}_q \neq \bar{m}_{q-1} \quad (289)$$

$$\tilde{l}_{1,i+1}^u + \tilde{r}_{1,i-1}^u - (\tilde{t}_{1i}^u + \tilde{e}_{1i}^d + \tilde{e}_{1i,3}^d) = t_{1i}^u + e_{1i}^d + e_{1i,3}^d \quad \forall i \in (I_0^1 \cup I_4^1) \setminus \{\underline{m}_1, \bar{m}_1\} \quad (290)$$

$$\tilde{l}_{1,\underline{m}_1+1}^u + \tilde{y}_u^0 - (\tilde{t}_{1,\underline{m}_1}^u + \tilde{e}_{1,\underline{m}_1,1}^d + \tilde{e}_{1,\underline{m}_1,3}^d) = t_{1,\underline{m}_1}^u + e_{1,\underline{m}_1,1}^d + e_{1,\underline{m}_1,3}^d \quad \text{if } \underline{m}_1 \in I_0^1 \cup I_4^1 \quad (291)$$

$$\tilde{r}_{1,\bar{m}_1-1}^u - (\tilde{t}_{1,\bar{m}_1}^u + \tilde{e}_{1,\bar{m}_1,1}^d + \tilde{e}_{1,\bar{m}_1,3}^d) = t_{1,\bar{m}_1}^u + e_{1,\bar{m}_1,1}^d + e_{1,\bar{m}_1,3}^d \quad \text{if } \bar{m}_1 \in I_0^1 \cup I_4^1 \quad (292)$$

$$\tilde{l}_{q,i+1}^u + \tilde{r}_{q,i-1}^u + \tilde{v}_{q-1,i}^u - (\tilde{t}_{qi}^u + \tilde{e}_{qi}^d + \tilde{e}_{qi,3}^d) = t_{qi}^u + e_{qi}^d + e_{qi,3}^d \quad \forall q \in B \setminus \{1\}, i \in (I_0^q \cup I_4^q) \setminus \{\underline{m}_q, \bar{m}_q\} \quad (293)$$

$$\tilde{l}_{q,\underline{m}_q+1}^u + \tilde{v}_{q-1,\underline{m}_q}^u - (\tilde{r}_{q,\underline{m}_q}^u + \tilde{e}_{q,\underline{m}_q,1}^d + \tilde{e}_{q,\underline{m}_q,3}^d) = t_{q,\underline{m}_q}^u + e_{q,\underline{m}_q,1}^d + e_{q,\underline{m}_q,3}^d \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_0^q \cup I_4^q \text{ and } \underline{m}_q = \underline{m}_{q-1} \quad (294)$$

$$\tilde{r}_{q,\underline{m}_q-1}^u + \tilde{l}_{q,\underline{m}_q+1}^u + \tilde{v}_{q-1,\underline{m}_q}^u - (\tilde{r}_{q,\underline{m}_q}^u + \tilde{e}_{q,\underline{m}_q,1}^d + \tilde{e}_{q,\underline{m}_q,3}^d) = t_{q,\underline{m}_q}^u + e_{q,\underline{m}_q,1}^d + e_{q,\underline{m}_q,3}^d \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_0^q \cup I_4^q \text{ and } \underline{m}_q \neq \underline{m}_{q-1} \quad (295)$$

$$\tilde{r}_{q,\bar{m}_q-1}^u + \tilde{v}_{q-1,\bar{m}_q}^u - (\tilde{r}_{q,\bar{m}_q}^u + \tilde{e}_{q,\bar{m}_q,1}^d + \tilde{e}_{q,\bar{m}_q,3}^d) = t_{q,\bar{m}_q}^u + e_{q,\bar{m}_q,1}^d + e_{q,\bar{m}_q,3}^d \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_0^q \cup I_4^q \text{ and } \bar{m}_q = \bar{m}_{q-1} \quad (296)$$

$$\tilde{r}_{q,\bar{m}_q-1}^u + \tilde{l}_{q,\bar{m}_q+1}^u + \tilde{v}_{q-1,\bar{m}_q}^u - (\tilde{r}_{q,\bar{m}_q}^u + \tilde{e}_{q,\bar{m}_q,1}^d + \tilde{e}_{q,\bar{m}_q,3}^d) = t_{q,\bar{m}_q}^u + e_{q,\bar{m}_q,1}^d + e_{q,\bar{m}_q,3}^d \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_0^q \cup I_4^q \text{ and } \bar{m}_q \neq \bar{m}_{q-1} \quad (297)$$

$$\tilde{l}_{1,i+1}^u + \tilde{r}_{1,i-1}^u - (\tilde{r}_{1,i}^u + \tilde{e}_{1i}^d + \tilde{e}_{1i2}^d) = t_{1i}^u + e_{1i}^d + e_{1i2}^d \quad \forall i \in (I_3^1 \cup I_5^1) \setminus \{\underline{m}_1, \bar{m}_1\} \quad (298)$$

$$\tilde{l}_{1,\underline{m}_1+1}^u + \tilde{v}_u^0 - (\tilde{r}_{1,\underline{m}_1}^u + \tilde{e}_{1,\underline{m}_1,1}^d + \tilde{e}_{1,\underline{m}_1,2}^d) = t_{1,\underline{m}_1}^u + e_{1,\underline{m}_1,1}^d + e_{1,\underline{m}_1,2}^d \quad \text{if } \underline{m}_1 \in I_3^1 \cup I_5^1 \quad (299)$$

$$\tilde{r}_{1,\bar{m}_1-1}^u - (\tilde{r}_{1,\bar{m}_1}^u + \tilde{e}_{1,\bar{m}_1,1}^d + \tilde{e}_{1,\bar{m}_1,2}^d) = t_{1,\bar{m}_1}^u + e_{1,\bar{m}_1,1}^d + e_{1,\bar{m}_1,2}^d \quad \text{if } \bar{m}_1 \in I_3^1 \cup I_5^1 \quad (300)$$

$$\tilde{l}_{q,i+1}^u + \tilde{r}_{q,i-1}^u + \tilde{v}_{q-1,i}^u - (\tilde{r}_{qi}^u + \tilde{e}_{qi1}^d + \tilde{e}_{qi2}^d) = t_{qi}^u + e_{qi1}^d + e_{qi2}^d \quad \forall q \in B \setminus \{1\}, i \in (I_3^q \cup I_5^q) \setminus \{\underline{m}_q, \bar{m}_q\} \quad (301)$$

$$\tilde{l}_{q,\underline{m}_q+1}^u + \tilde{v}_{q-1,\underline{m}_q}^u - (\tilde{r}_{q,\underline{m}_q}^u + \tilde{e}_{q,\underline{m}_q,1}^d + \tilde{e}_{q,\underline{m}_q,2}^d) = t_{q,\underline{m}_q}^u + e_{q,\underline{m}_q,1}^d + e_{q,\underline{m}_q,2}^d \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_3^q \cup I_5^q \text{ and } \underline{m}_q = \underline{m}_{q-1} \quad (302)$$

$$\tilde{r}_{q,\underline{m}_q-1}^u + \tilde{l}_{q,\underline{m}_q+1}^u + \tilde{v}_{q-1,\underline{m}_q}^u - (\tilde{r}_{q,\underline{m}_q}^u + \tilde{e}_{q,\underline{m}_q,1}^d + \tilde{e}_{q,\underline{m}_q,2}^d) = t_{q,\underline{m}_q}^u + e_{q,\underline{m}_q,1}^d + e_{q,\underline{m}_q,2}^d \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_3^q \cup I_5^q \text{ and } \underline{m}_q \neq \underline{m}_{q-1} \quad (303)$$

$$\tilde{r}_{q,\bar{m}_q-1}^u + \tilde{v}_{q-1,\bar{m}_q}^u - (\tilde{r}_{q,\bar{m}_q}^u + \tilde{e}_{q,\bar{m}_q,1}^d + \tilde{e}_{q,\bar{m}_q,2}^d) = t_{q,\bar{m}_q}^u + e_{q,\bar{m}_q,1}^d + e_{q,\bar{m}_q,2}^d \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_3^q \cup I_5^q \text{ and } \bar{m}_q = \bar{m}_{q-1} \quad (304)$$

$$\tilde{r}_{q,\bar{m}_q-1}^u + \tilde{l}_{q,\bar{m}_q+1}^u + \tilde{v}_{q-1,\bar{m}_q}^u - (\tilde{r}_{q,\bar{m}_q}^u + \tilde{e}_{q,\bar{m}_q,1}^d + \tilde{e}_{q,\bar{m}_q,2}^d) = t_{q,\bar{m}_q}^u + e_{q,\bar{m}_q,1}^d + e_{q,\bar{m}_q,2}^d \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_3^q \cup I_5^q \text{ and } \bar{m}_q \neq \bar{m}_{q-1} \quad (305)$$

- Constraints corresponding to vertices $[d, q, i]$

$$\tilde{l}_{2,i+1}^d + \tilde{r}_{2,i-1}^d + \tilde{v}_{2i}^d - (\tilde{v}_{1i}^{d,l} + \tilde{v}_{1i}^{d,r}) = v_{1i}^{d,l} + v_{1i}^{d,r} \quad \forall i \in I_1^1 \setminus \{\underline{m}_1, \bar{m}_1\} \text{ with } \underline{m}_2 \leq i \leq \bar{m}_2 \quad (306)$$

$$\tilde{l}_{2,i+1}^d + \tilde{r}_{2,i-1}^d - (\tilde{v}_{1i}^{d,l} + \tilde{v}_{1i}^{d,r}) = v_{1i}^{d,l} + v_{1i}^{d,r} \quad \forall i \in I_1^1 \setminus \{\underline{m}_1, \bar{m}_1\} \text{ with } i < \underline{m}_2 \text{ or } i > \bar{m}_2 \quad (307)$$

$$\tilde{l}_{2,\underline{m}_1+1}^d + \tilde{v}_{2,\underline{m}_1}^d - (\tilde{v}_{1,\underline{m}_1}^{d,l} + \tilde{v}_{1,\underline{m}_1}^{d,r}) = v_{1,\underline{m}_1}^{d,l} + v_{1,\underline{m}_1}^{d,r} \quad \text{if } \underline{m}_1 \in I_1^1 \text{ and } \underline{m}_1 = \underline{m}_2 \quad (308)$$

$$\tilde{l}_{2,\underline{m}_1+1}^d - (\tilde{v}_{1,\underline{m}_1}^{d,l} + \tilde{v}_{1,\underline{m}_1}^{d,r}) = v_{1,\underline{m}_1}^{d,l} + v_{1,\underline{m}_1}^{d,r} \quad \text{if } \underline{m}_1 \in I_1^1 \text{ and } \underline{m}_1 \neq \underline{m}_2 \quad (309)$$

$$\tilde{r}_{2,\bar{m}_1-1}^d + \tilde{v}_{2,\bar{m}_1}^d - \tilde{v}_{1,\bar{m}_1}^{d,l} = v_{1,\bar{m}_1}^{d,l} \quad \text{if } \bar{m}_1 \in I_1^1 \text{ and } \bar{m}_1 = \bar{m}_2 \quad (310)$$

$$\tilde{r}_{2,\bar{m}_1-1}^d - \tilde{v}_{1,\bar{m}_1}^{d,l} = v_{1,\bar{m}_1}^{d,l} \quad \text{if } \bar{m}_1 \in I_1^1 \text{ and } \bar{m}_1 \neq \bar{m}_2 \quad (311)$$

$$\tilde{l}_{q+1,i+1}^d + \tilde{r}_{q+1,i-1}^d + \tilde{v}_{q+1,i}^d - (\tilde{v}_{qi}^{d,l} + \tilde{v}_{qi}^{d,r} + \tilde{v}_{qi}^d) = v_{qi}^{d,l} + v_{qi}^{d,r} + v_{qi}^d \quad \forall q \in B \setminus \{1, p\}, i \in I_1^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} \quad (312)$$

$$\tilde{l}_{q+1,i+1}^d + \tilde{r}_{q+1,i-1}^d - (\tilde{v}_{qi}^{d,l} + \tilde{v}_{qi}^{d,r} + \tilde{v}_{qi}^d) = v_{qi}^{d,l} + v_{qi}^{d,r} + v_{qi}^d \quad \forall q \in B \setminus \{1, p\}, i \in I_1^q \setminus \{\underline{m}_q, \bar{m}_q\}, \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (313)$$

$$\tilde{l}_{q+1, \underline{m}_q+1}^d + \tilde{v}_{q+1, \underline{m}_q}^d - (\tilde{v}_{q, \underline{m}_q}^{d,r} + \tilde{v}_{q, \underline{m}_q}^d) = v_{q, \underline{m}_q}^{d,r} + v_{q, \underline{m}_q}^d \quad \forall q \in B \setminus \{1, p\} \text{ with } \underline{m}_q \in I_1^q \text{ and } \underline{m}_q = \underline{m}_{q+1}, \underline{m}_q = \underline{m}_{q-1} \quad (314)$$

$$\tilde{l}_{q+1, \underline{m}_q+1}^d - (\tilde{v}_{q, \underline{m}_q}^{d,r} + \tilde{v}_{q, \underline{m}_q}^d) = v_{q, \underline{m}_q}^{d,r} + v_{q, \underline{m}_q}^d \quad \forall q \in B \setminus \{1, p\} \text{ with } \underline{m}_q \in I_1^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1}, \underline{m}_q = \underline{m}_{q-1} \quad (315)$$

$$\tilde{r}_{q+1, \bar{m}_q-1}^d + \tilde{v}_{q+1, \bar{m}_q}^d - (\tilde{v}_{q, \bar{m}_q}^{d,l} + \tilde{v}_{q, \bar{m}_q}^d) = v_{q, \bar{m}_q}^{d,l} + v_{q, \bar{m}_q}^d \quad \forall q \in B \setminus \{1, p\} \text{ with } \bar{m}_q \in I_1^q \text{ and } \bar{m}_q = \bar{m}_{q+1}, \bar{m}_q = \bar{m}_{q-1} \quad (316)$$

$$\tilde{r}_{q+1, \bar{m}_q-1}^d - (\tilde{v}_{q, \bar{m}_q}^{d,l} + \tilde{v}_{q, \bar{m}_q}^d) = v_{q, \bar{m}_q}^{d,l} + v_{q, \bar{m}_q}^d \quad \forall q \in B \setminus \{1, p\} \text{ with } \bar{m}_q \in I_1^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1}, \bar{m}_q = \bar{m}_{q-1} \quad (317)$$

$$\tilde{l}_{p+1, i+1}^d + \tilde{r}_{p+1, i-1}^d - (\tilde{v}_{pi}^{d,l} + \tilde{v}_{pi}^{d,r} + \tilde{v}_{pi}^d) = v_{pi}^{d,l} + v_{pi}^{d,r} + v_{pi}^d \quad \forall i \in I_1^p \setminus \{\underline{m}_p, \bar{m}_p\} \quad (318)$$

$$\tilde{l}_{p+1, \underline{m}_p+1}^d - (\tilde{v}_{p, \underline{m}_p}^{d,l} + \tilde{v}_{p, \underline{m}_p}^{d,r} + \tilde{v}_{p, \underline{m}_p}^d) = v_{p, \underline{m}_p}^{d,l} + v_{p, \underline{m}_p}^{d,r} + v_{p, \underline{m}_p}^d \quad \text{if } \underline{m}_p \in I_1^p \text{ and } \underline{m}_p \neq \underline{m}_{p-1} \quad (319)$$

$$\tilde{l}_{p+1, \underline{m}_p+1}^d - (\tilde{v}_{p, \underline{m}_p}^{d,r} + \tilde{v}_{p, \underline{m}_p}^d) = v_{p, \underline{m}_p}^{d,r} + v_{p, \underline{m}_p}^d \quad \text{if } \underline{m}_p \in I_1^p \text{ and } \underline{m}_p = \underline{m}_{p-1} \quad (320)$$

$$\tilde{r}_{p+1, \bar{m}_p-1}^d - (\tilde{v}_{p, \bar{m}_p}^{d,l} + \tilde{v}_{p, \bar{m}_p}^{d,r} + \tilde{v}_{p, \bar{m}_p}^d) = v_{p, \bar{m}_p}^{d,l} + v_{p, \bar{m}_p}^{d,r} + v_{p, \bar{m}_p}^d \quad \text{if } \bar{m}_p \in I_1^p \text{ and } \bar{m}_p \neq \bar{m}_{p-1} \quad (321)$$

$$\tilde{r}_{p+1, \bar{m}_p-1}^d - (\tilde{v}_{p, \bar{m}_p}^{d,l} + \tilde{v}_{p, \bar{m}_p}^d) = v_{p, \bar{m}_p}^{d,l} + v_{p, \bar{m}_p}^d \quad \text{if } \bar{m}_p \in I_1^p \text{ and } \bar{m}_p = \bar{m}_{p-1} \quad (322)$$

$$\tilde{l}_{q+1, \underline{m}_q+1}^d + \tilde{v}_{q+1, \underline{m}_q}^d - (\tilde{v}_{q, \underline{m}_q}^{d,l} + \tilde{v}_{q, \underline{m}_q}^{d,r} + \tilde{v}_{q, \underline{m}_q}^d) = v_{q, \underline{m}_q}^{d,l} + v_{q, \underline{m}_q}^{d,r} + v_{q, \underline{m}_q}^d \quad \forall q \in B \setminus \{1, p\} \text{ with } \underline{m}_q \in I_1^q \text{ and } \underline{m}_q = \underline{m}_{q+1}, \underline{m}_q \neq \underline{m}_{q-1} \quad (323)$$

$$\tilde{l}_{q+1, \underline{m}_q+1}^d - (\tilde{v}_{q, \underline{m}_q}^{d,l} + \tilde{v}_{q, \underline{m}_q}^{d,r} + \tilde{v}_{q, \underline{m}_q}^d) = v_{q, \underline{m}_q}^{d,l} + v_{q, \underline{m}_q}^{d,r} + v_{q, \underline{m}_q}^d \quad \forall q \in B \setminus \{1, p\} \text{ with } \underline{m}_q \in I_1^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1}, \underline{m}_q \neq \underline{m}_{q-1} \quad (324)$$

$$\tilde{r}_{q+1, \bar{m}_q-1}^d + \tilde{v}_{q+1, \bar{m}_q}^d - (\tilde{v}_{q, \bar{m}_q}^{d,l} + \tilde{v}_{q, \bar{m}_q}^{d,r} + \tilde{v}_{q, \bar{m}_q}^d) = v_{q, \bar{m}_q}^{d,l} + v_{q, \bar{m}_q}^{d,r} + v_{q, \bar{m}_q}^d \quad \forall q \in B \setminus \{1, p\} \text{ with } \bar{m}_q \in I_1^q \text{ and } \bar{m}_q = \bar{m}_{q+1}, \bar{m}_q \neq \bar{m}_{q-1} \quad (325)$$

$$\tilde{r}_{q+1, \bar{m}_q-1}^d - (\tilde{v}_{q, \bar{m}_q}^{d,l} + \tilde{v}_{q, \bar{m}_q}^{d,r} + \tilde{v}_{q, \bar{m}_q}^d) = v_{q, \bar{m}_q}^{d,l} + v_{q, \bar{m}_q}^{d,r} + v_{q, \bar{m}_q}^d \quad \forall q \in B \setminus \{1, p\} \text{ with } \bar{m}_q \in I_1^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1}, \bar{m}_q \neq \bar{m}_{q-1} \quad (326)$$

$$\tilde{l}_{q+1, i+1}^d + \tilde{r}_{q+1, i-1}^d + \tilde{v}_{q+1, i}^d - (\tilde{e}_{qi1}^u + \tilde{t}_{qi}^d) = e_{qi1}^u + t_{qi}^d \quad \forall q \in B \setminus \{p\}, i \in I_2^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} \quad (327)$$

$$\tilde{l}_{q+1, i+1}^d + \tilde{r}_{q+1, i-1}^d - (\tilde{e}_{qi1}^u + \tilde{t}_{qi}^d) = e_{qi1}^u + t_{qi}^d \quad \forall q \in B \setminus \{p\}, i \in I_2^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (328)$$

$$\tilde{l}_{q+1, \underline{m}_q+1}^d + \tilde{v}_{q+1, \underline{m}_q}^d - (\tilde{e}_{q, \underline{m}_q, 1}^u + \tilde{t}_{q, \underline{m}_q}^d) = e_{q, \underline{m}_q, 1}^u + t_{q, \underline{m}_q}^d \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_2^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \quad (329)$$

$$\tilde{l}_{q+1, \underline{m}_q+1}^d - (\tilde{e}_{q, \underline{m}_q, 1}^u + \tilde{t}_{q, \underline{m}_q}^d) = e_{q, \underline{m}_q, 1}^u + t_{q, \underline{m}_q}^d \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_2^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} \quad (330)$$

$$\tilde{r}_{q+1, \bar{m}_q-1}^d + \tilde{v}_{q+1, \bar{m}_q}^d - (\tilde{e}_{q, \bar{m}_q, 1}^u + \tilde{t}_{q, \bar{m}_q}^d) = e_{q, \bar{m}_q, 1}^u + t_{q, \bar{m}_q}^d \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_2^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \quad (331)$$

$$\tilde{r}_{q+1, \bar{m}_q-1}^d - (\tilde{e}_{q, \bar{m}_q, 1}^u + \tilde{t}_{q, \bar{m}_q}^d) = e_{q, \bar{m}_q, 1}^u + t_{q, \bar{m}_q}^d \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_2^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \quad (332)$$

$$\tilde{l}_{p+1, i+1}^d + \tilde{r}_{p+1, i-1}^d - (\tilde{e}_{pi1}^u + \tilde{t}_{pi}^d) = e_{pi1}^u + t_{pi}^d \quad \forall i \in I_2^p \setminus \{\underline{m}_p, \bar{m}_p\} \quad (333)$$

$$\tilde{l}_{p+1, \underline{m}_p+1}^d - (\tilde{e}_{p, \underline{m}_p, 1}^u + \tilde{t}_{p, \underline{m}_p}^d) = e_{p, \underline{m}_p, 1}^u + t_{p, \underline{m}_p}^d \quad \text{if } \underline{m}_p \in I_2^p \quad (334)$$

$$\tilde{r}_{p+1, \bar{m}_p-1}^d - (\tilde{e}_{p, \bar{m}_p, 1}^u + \tilde{t}_{p, \bar{m}_p}^d) = e_{p, \bar{m}_p, 1}^u + t_{p, \bar{m}_p}^d \quad \text{if } \bar{m}_p \in I_2^p \quad (335)$$

$$\tilde{l}_{q+1, i+1}^d + \tilde{r}_{q+1, i-1}^d + \tilde{v}_{q+1, i}^d - (\tilde{e}_{qi1}^u + \tilde{e}_{qi2}^u + \tilde{t}_{qi}^d) = e_{qi1}^u + e_{qi2}^u + t_{qi}^d \quad \forall q \in B \setminus \{p\}, i \in (I_3^q \cup I_4^q) \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} \quad (336)$$

$$\tilde{l}_{q+1, i+1}^d + \tilde{r}_{q+1, i-1}^d - (\tilde{e}_{qi1}^u + \tilde{e}_{qi2}^u + \tilde{t}_{qi}^d) = e_{qi1}^u + e_{qi2}^u + t_{qi}^d \quad \forall q \in B \setminus \{p\}, i \in (I_3^q \cup I_4^q) \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (337)$$

$$\tilde{l}_{q+1, \underline{m}_q+1}^d + \tilde{v}_{q+1, \underline{m}_q}^d - (\tilde{e}_{q, \underline{m}_q, 1}^u + \tilde{e}_{q, \underline{m}_q, 2}^u + \tilde{t}_{q, \underline{m}_q}^d) = e_{q, \underline{m}_q, 1}^u + e_{q, \underline{m}_q, 2}^u + t_{q, \underline{m}_q}^d \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_3^q \cup I_4^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \quad (338)$$

$$\tilde{l}_{q+1,\underline{m}_q+1}^d - (\tilde{e}_{q,\underline{m}_q,1}^u + \tilde{e}_{q,\underline{m}_q,2}^u + \tilde{t}_{q,\underline{m}_q}^d) = e_{q,\underline{m}_q,1}^u + e_{q,\underline{m}_q,2}^u + t_{q,\underline{m}_q}^d \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_3^q \cup I_4^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} \quad (339)$$

$$\tilde{r}_{q+1,\bar{m}_q-1}^d + \tilde{v}_{q+1,\bar{m}_q}^d - (\tilde{e}_{q,\bar{m}_q,1}^u + \tilde{e}_{q,\bar{m}_q,2}^u + \tilde{t}_{q,\bar{m}_q}^d) = e_{q,\bar{m}_q,1}^u + e_{q,\bar{m}_q,2}^u + t_{q,\bar{m}_q}^d \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_3^q \cup I_4^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \quad (340)$$

$$\tilde{r}_{q+1,\bar{m}_q-1}^d - (\tilde{e}_{q,\bar{m}_q,1}^u + \tilde{e}_{q,\bar{m}_q,2}^u + \tilde{t}_{q,\bar{m}_q}^d) = e_{q,\bar{m}_q,1}^u + e_{q,\bar{m}_q,2}^u + t_{q,\bar{m}_q}^d \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_3^q \cup I_4^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \quad (341)$$

$$\tilde{l}_{p+1,i+1}^d + \tilde{r}_{p+1,i-1}^d - (\tilde{e}_{p,i}^u + \tilde{e}_{p,i}^u + \tilde{t}_{p,i}^d) = e_{p,i}^u + e_{p,i}^u + t_{p,i}^d \quad \forall i \in (I_3^p \cup I_4^p) \setminus \{\underline{m}_p, \bar{m}_p\} \quad (342)$$

$$\tilde{l}_{p+1,\underline{m}_p+1}^d - (\tilde{e}_{p,\underline{m}_p,1}^u + \tilde{e}_{p,\underline{m}_p,2}^u + \tilde{t}_{p,\underline{m}_p}^d) = e_{p,\underline{m}_p,1}^u + e_{p,\underline{m}_p,2}^u + t_{p,\underline{m}_p}^d \quad \text{if } \underline{m}_p \in I_3^p \cup I_4^p \quad (343)$$

$$\tilde{r}_{p+1,\bar{m}_p-1}^d - (\tilde{e}_{p,\bar{m}_p,1}^u + \tilde{e}_{p,\bar{m}_p,2}^u + \tilde{t}_{p,\bar{m}_p}^d) = e_{p,\bar{m}_p,1}^u + e_{p,\bar{m}_p,2}^u + t_{p,\bar{m}_p}^d \quad \text{if } \bar{m}_p \in I_3^p \cup I_4^p \quad (344)$$

$$\tilde{l}_{q+1,i+1}^d + \tilde{r}_{q+1,i-1}^d + \tilde{v}_{q+1,i}^d - (\tilde{e}_{q,i}^u + \tilde{e}_{q,i}^u + \tilde{t}_{q,i}^d) = e_{q,i}^u + e_{q,i}^u + t_{q,i}^d \quad \forall q \in B \setminus \{p\}, i \in (I_0^q \cup I_5^q) \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} \quad (345)$$

$$\tilde{l}_{q+1,i+1}^d + \tilde{r}_{q+1,i-1}^d - (\tilde{e}_{q,i}^u + \tilde{e}_{q,i}^u + \tilde{t}_{q,i}^d) = e_{q,i}^u + e_{q,i}^u + t_{q,i}^d \quad \forall q \in B \setminus \{p\}, i \in (I_0^q \cup I_5^q) \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (346)$$

$$\tilde{l}_{q+1,\underline{m}_q+1}^d + \tilde{v}_{q+1,\underline{m}_q}^d - (\tilde{e}_{q,\underline{m}_q,1}^u + \tilde{e}_{q,\underline{m}_q,3}^u + \tilde{t}_{q,\underline{m}_q}^d) = e_{q,\underline{m}_q,1}^u + e_{q,\underline{m}_q,3}^u + t_{q,\underline{m}_q}^d \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_0^q \cup I_5^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \quad (347)$$

$$\tilde{l}_{q+1,\underline{m}_q+1}^d - (\tilde{e}_{q,\underline{m}_q,1}^u + \tilde{e}_{q,\underline{m}_q,3}^u + \tilde{t}_{q,\underline{m}_q}^d) = e_{q,\underline{m}_q,1}^u + e_{q,\underline{m}_q,3}^u + t_{q,\underline{m}_q}^d \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_0^q \cup I_5^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} \quad (348)$$

$$\tilde{r}_{q+1,\bar{m}_q-1}^d + \tilde{v}_{q+1,\bar{m}_q}^d - (\tilde{e}_{q,\bar{m}_q,1}^u + \tilde{e}_{q,\bar{m}_q,3}^u + \tilde{t}_{q,\bar{m}_q}^d) = e_{q,\bar{m}_q,1}^u + e_{q,\bar{m}_q,3}^u + t_{q,\bar{m}_q}^d \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_0^q \cup I_5^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \quad (349)$$

$$\tilde{r}_{q+1,\bar{m}_q-1}^d - (\tilde{e}_{q,\bar{m}_q,1}^u + \tilde{e}_{q,\bar{m}_q,3}^u + \tilde{t}_{q,\bar{m}_q}^d) = e_{q,\bar{m}_q,1}^u + e_{q,\bar{m}_q,3}^u + t_{q,\bar{m}_q}^d \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_0^q \cup I_5^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \quad (350)$$

$$\tilde{l}_{p+1,i+1}^d + \tilde{r}_{p+1,i-1}^d - (\tilde{e}_{p,i}^u + \tilde{e}_{p,i}^u + \tilde{t}_{p,i}^d) = e_{p,i}^u + e_{p,i}^u + t_{p,i}^d \quad \forall i \in (I_0^p \cup I_5^p) \setminus \{\underline{m}_p, \bar{m}_p\} \quad (351)$$

$$\tilde{l}_{p+1,\underline{m}_p+1}^d - (\tilde{e}_{p,\underline{m}_p,1}^u + \tilde{e}_{p,\underline{m}_p,3}^u + \tilde{t}_{p,\underline{m}_p}^d) = e_{p,\underline{m}_p,1}^u + e_{p,\underline{m}_p,3}^u + t_{p,\underline{m}_p}^d \quad \text{if } \underline{m}_p \in I_0^p \cup I_5^p \quad (352)$$

$$\tilde{r}_{p+1,\bar{m}_p-1}^d - (\tilde{e}_{p,\bar{m}_p,1}^u + \tilde{e}_{p,\bar{m}_p,3}^u + \tilde{t}_{p,\bar{m}_p}^d) = e_{p,\bar{m}_p,1}^u + e_{p,\bar{m}_p,3}^u + t_{p,\bar{m}_p}^d \quad \text{if } \bar{m}_p \in I_0^p \cup I_5^p \quad (353)$$

- Constraints corresponding to vertices $[u, q, i, 1]$

$$\tilde{e}_{qi}^u + \tilde{t}_{qi}^u - (\tilde{v}_{qi}^u + \tilde{v}_{qi}^{u,l} + \tilde{v}_{qi}^{u,r}) = v_{qi}^u + v_{qi}^{u,l} + v_{qi}^{u,r} \quad \forall q \in B \setminus \{p\}, i \in I_2^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} \quad (354)$$

$$\tilde{e}_{qi}^u + \tilde{t}_{qi}^u - (\tilde{v}_{qi}^{u,l} + \tilde{v}_{qi}^{u,r}) = v_{qi}^{u,l} + v_{qi}^{u,r} \quad \forall q \in B \setminus \{p\}, i \in I_2^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (355)$$

$$\tilde{e}_{q,\underline{m}_q,1}^u + \tilde{t}_{q,\underline{m}_q}^u - (\tilde{v}_{q,\underline{m}_q}^u + \tilde{v}_{q,\underline{m}_q}^{u,r}) = v_{q,\underline{m}_q}^u + v_{q,\underline{m}_q}^{u,r} \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_2^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \quad (356)$$

$$\tilde{e}_{q,\underline{m}_q,1}^u + \tilde{t}_{q,\underline{m}_q}^u - \tilde{v}_{q,\underline{m}_q}^{u,r} = v_{q,\underline{m}_q}^{u,r} \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_2^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} \quad (357)$$

$$\tilde{e}_{q,\bar{m}_q,1}^u + \tilde{t}_{q,\bar{m}_q}^u - (\tilde{v}_{q,\bar{m}_q}^u + \tilde{v}_{q,\bar{m}_q}^{u,l}) = v_{q,\bar{m}_q}^u + v_{q,\bar{m}_q}^{u,l} \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_2^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \quad (358)$$

$$\tilde{e}_{q,\bar{m}_q,1}^u + \tilde{t}_{q,\bar{m}_q}^u - \tilde{v}_{q,\bar{m}_q}^{u,l} = v_{q,\bar{m}_q}^{u,l} \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_2^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \quad (359)$$

$$\tilde{e}_{pi}^u + \tilde{t}_{pi}^u - (\tilde{v}_{pi}^{u,l} + \tilde{v}_{pi}^{u,r}) = v_{pi}^{u,l} + v_{pi}^{u,r} \quad \forall i \in I_2^p \setminus \{\underline{m}_p, \bar{m}_p\} \quad (360)$$

$$\tilde{e}_{p,\underline{m}_p,1}^u + \tilde{t}_{p,\underline{m}_p}^u - \tilde{v}_{p,\underline{m}_p}^{u,r} = v_{p,\underline{m}_p}^{u,r} \quad \text{if } \underline{m}_p \in I_2^p \quad (361)$$

$$\tilde{e}_{p,\bar{m}_p,1}^u + \tilde{t}_{p,\bar{m}_p}^u - \tilde{v}_{p,\bar{m}_p}^{u,l} = v_{p,\bar{m}_p}^{u,l} \quad \text{if } \bar{m}_p \in I_2^p \quad (362)$$

$$\tilde{e}_{qi}^u + \tilde{t}_{qi}^u - \tilde{w}_{qi}^u = w_{qi}^u \quad \forall q \in B, i \in I^q \setminus (I_1^q \cup I_2^q) \quad (363)$$

- Constraints corresponding to vertices $[u, q, i, 2]$

$$\tilde{e}_{qi2}^u + \tilde{w}_{qi1}^u - (\tilde{v}_{qi}^u + \tilde{v}_{qi}^{u,l} + \tilde{v}_{qi}^{u,r}) = v_{qi}^u + v_{qi}^{u,l} + v_{qi}^{u,r}$$

$$\tilde{e}_{qi2}^u + \tilde{w}_{qi1}^u - (\tilde{v}_{qi}^{u,l} + \tilde{v}_{qi}^{u,r}) = v_{qi}^{u,l} + v_{qi}^{u,r}$$

$$\tilde{e}_{q,\underline{m}_q,2}^u + \tilde{w}_{q,\underline{m}_q,1}^u - (\tilde{v}_{q,\underline{m}_q}^u + \tilde{v}_{q,\underline{m}_q}^{u,r}) = v_{q,\underline{m}_q}^u + v_{q,\underline{m}_q}^{u,r}$$

$$\tilde{e}_{q,\underline{m}_q,2}^u + \tilde{w}_{q,\underline{m}_q,1}^u - \tilde{v}_{q,\underline{m}_q}^{u,r} = v_{q,\underline{m}_q}^{u,r}$$

$$\tilde{e}_{q,\bar{m}_q,2}^u + \tilde{w}_{q,\bar{m}_q,1}^u - (\tilde{v}_{q,\bar{m}_q}^u + \tilde{v}_{q,\bar{m}_q}^{u,l}) = v_{q,\bar{m}_q}^u + v_{q,\bar{m}_q}^{u,l}$$

$$\tilde{e}_{q,\bar{m}_q,2}^u + \tilde{w}_{q,\bar{m}_q,1}^u - \tilde{v}_{q,\bar{m}_q}^{u,l} = v_{q,\bar{m}_q}^{u,l}$$

$$\tilde{e}_{pi2}^u + \tilde{w}_{pi1}^u - (\tilde{v}_{pi}^{u,l} + \tilde{v}_{pi}^{u,r}) = v_{pi}^{u,l} + v_{pi}^{u,r}$$

$$\tilde{e}_{p,\underline{m}_p,2}^u + \tilde{w}_{p,\underline{m}_p,1}^u - \tilde{v}_{p,\underline{m}_p}^{u,r} = v_{p,\underline{m}_p}^{u,r}$$

$$\tilde{e}_{p,\bar{m}_p,2}^u + \tilde{w}_{p,\bar{m}_p,1}^u - \tilde{v}_{p,\bar{m}_p}^{u,l} = v_{p,\bar{m}_p}^{u,l}$$

$$\tilde{e}_{qi2}^u + \tilde{w}_{qi1}^u - \tilde{w}_{qi2}^u = w_{qi2}^u$$

$$\tilde{w}_{qi1}^u - \tilde{w}_{qi2}^u = w_{qi2}^u$$

$$\forall q \in B \setminus \{p\}, i \in I_3^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} \quad (364)$$

$$\forall q \in B \setminus \{p\}, i \in I_3^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (365)$$

$$\forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_3^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \quad (366)$$

$$\forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_3^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} \quad (367)$$

$$\forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_3^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \quad (368)$$

$$\forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_3^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \quad (369)$$

$$\forall i \in I_3^p \setminus \{\underline{m}_p, \bar{m}_p\} \quad (370)$$

$$\text{if } \underline{m}_p \in I_3^p \quad (371)$$

$$\text{if } \bar{m}_p \in I_3^p \quad (372)$$

$$\forall q \in B, i \in I_4^q \quad (373)$$

$$\forall q \in B, i \in I_0^q \cup I_5^q \quad (374)$$

- Constraints corresponding to vertices $[u, q, i, 3]$

$$\tilde{w}_{qi2}^u - (\tilde{v}_{qi}^u + \tilde{v}_{qi}^{u,l} + \tilde{v}_{qi}^{u,r}) = v_{qi}^u + v_{qi}^{u,l} + v_{qi}^{u,r}$$

$$\tilde{w}_{qi2}^u - (\tilde{v}_{qi}^{u,l} + \tilde{v}_{qi}^{u,r}) = v_{qi}^{u,l} + v_{qi}^{u,r}$$

$$\tilde{w}_{q,\underline{m}_q,2}^u - (\tilde{v}_{q,\underline{m}_q}^u + \tilde{v}_{q,\underline{m}_q}^{u,r}) = v_{q,\underline{m}_q}^u + v_{q,\underline{m}_q}^{u,r}$$

$$\tilde{w}_{q,\underline{m}_q,2}^u - \tilde{v}_{q,\underline{m}_q}^{u,r} = v_{q,\underline{m}_q}^{u,r}$$

$$\tilde{w}_{q,\bar{m}_q,2}^u - (\tilde{v}_{q,\bar{m}_q}^u + \tilde{v}_{q,\bar{m}_q}^{u,l}) = v_{q,\bar{m}_q}^u + v_{q,\bar{m}_q}^{u,l}$$

$$\tilde{w}_{q,\bar{m}_q,2}^u - \tilde{v}_{q,\bar{m}_q}^{u,l} = v_{q,\bar{m}_q}^{u,l}$$

$$\tilde{w}_{pi2}^u - (\tilde{v}_{pi}^{u,l} + \tilde{v}_{pi}^{u,r}) = v_{pi}^{u,l} + v_{pi}^{u,r}$$

$$\tilde{w}_{p,\underline{m}_p,2}^u - \tilde{v}_{p,\underline{m}_p}^{u,r} = v_{p,\underline{m}_p}^{u,r}$$

$$\tilde{w}_{p,\bar{m}_p,2}^u - \tilde{v}_{p,\bar{m}_p}^{u,l} = v_{p,\bar{m}_p}^{u,l}$$

$$\tilde{e}_{qi3}^u + \tilde{w}_{qi2}^u - (\tilde{v}_{qi}^u + \tilde{v}_{qi}^{u,l} + \tilde{v}_{qi}^{u,r}) = v_{qi}^u + v_{qi}^{u,l} + v_{qi}^{u,r}$$

$$\tilde{e}_{qi3}^u + \tilde{w}_{qi2}^u - (\tilde{v}_{qi}^{u,l} + \tilde{v}_{qi}^{u,r}) = v_{qi}^{u,l} + v_{qi}^{u,r}$$

$$\tilde{e}_{q,\underline{m}_q,3}^u + \tilde{w}_{q,\underline{m}_q,2}^u - (\tilde{v}_{q,\underline{m}_q}^u + \tilde{v}_{q,\underline{m}_q}^{u,r}) = v_{q,\underline{m}_q}^u + v_{q,\underline{m}_q}^{u,r}$$

$$\tilde{e}_{q,\underline{m}_q,3}^u + \tilde{w}_{q,\underline{m}_q,2}^u - \tilde{v}_{q,\underline{m}_q}^{u,r} = v_{q,\underline{m}_q}^{u,r}$$

$$\tilde{e}_{q,\bar{m}_q,3}^u + \tilde{w}_{q,\bar{m}_q,2}^u - (\tilde{v}_{q,\bar{m}_q}^u + \tilde{v}_{q,\bar{m}_q}^{u,l}) = v_{q,\bar{m}_q}^u + v_{q,\bar{m}_q}^{u,l}$$

$$\tilde{e}_{q,\bar{m}_q,3}^u + \tilde{w}_{q,\bar{m}_q,2}^u - \tilde{v}_{q,\bar{m}_q}^{u,l} = v_{q,\bar{m}_q}^{u,l}$$

$$\tilde{e}_{pi3}^u + \tilde{w}_{pi2}^u - (\tilde{v}_{pi}^{u,l} + \tilde{v}_{pi}^{u,r}) = v_{pi}^{u,l} + v_{pi}^{u,r}$$

$$\tilde{e}_{p,\underline{m}_p,3}^u + \tilde{w}_{p,\underline{m}_p,2}^u - \tilde{v}_{p,\underline{m}_p}^{u,r} = v_{p,\underline{m}_p}^{u,r}$$

$$\tilde{e}_{p,\bar{m}_p,3}^u + \tilde{w}_{p,\bar{m}_p,2}^u - \tilde{v}_{p,\bar{m}_p}^{u,l} = v_{p,\bar{m}_p}^{u,l}$$

$$\tilde{e}_{qi3}^u + \tilde{w}_{qi2}^u - \tilde{w}_{qi3}^u = w_{qi3}^u$$

$$\forall q \in B \setminus \{p\}, i \in I_4^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} \quad (375)$$

$$\forall q \in B \setminus \{p\}, i \in I_4^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (376)$$

$$\forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_4^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \quad (377)$$

$$\forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_4^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} \quad (378)$$

$$\forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_4^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \quad (379)$$

$$\forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_4^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \quad (380)$$

$$\forall i \in I_4^p \setminus \{\underline{m}_p, \bar{m}_p\} \quad (381)$$

$$\text{if } \underline{m}_p \in I_4^p \quad (382)$$

$$\text{if } \bar{m}_p \in I_4^p \quad (383)$$

$$\forall q \in B \setminus \{p\}, i \in I_5^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} \quad (384)$$

$$\forall q \in B \setminus \{p\}, i \in I_5^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (385)$$

$$\forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_5^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \quad (386)$$

$$\forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_5^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} \quad (387)$$

$$\forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_5^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \quad (388)$$

$$\forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_5^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \quad (389)$$

$$\forall i \in I_5^p \setminus \{\underline{m}_p, \bar{m}_p\} \quad (390)$$

$$\text{if } \underline{m}_p \in I_5^p \quad (391)$$

$$\text{if } \bar{m}_p \in I_5^p \quad (392)$$

$$\forall q \in B, i \in I_0^q \quad (393)$$

- Constraints corresponding to vertices $[u, q, i, 4]$

$$\tilde{w}_{qi3}^u - (\tilde{v}_{qi}^u + \tilde{v}_{qi}^{u,l} + \tilde{v}_{qi}^{u,r}) = v_{qi}^u + v_{qi}^{u,l} + v_{qi}^{u,r} \quad \forall q \in B \setminus \{p\}, i \in I_0^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } \underline{m}_{q+1} \leq i \leq \bar{m}_{q+1} \quad (394)$$

$$\tilde{w}_{qi3}^u - (\tilde{v}_{qi}^{u,l} + \tilde{v}_{qi}^{u,r}) = v_{qi}^{u,l} + v_{qi}^{u,r} \quad \forall q \in B \setminus \{p\}, i \in I_0^q \setminus \{\underline{m}_q, \bar{m}_q\} \text{ with } i < \underline{m}_{q+1} \text{ or } i > \bar{m}_{q+1} \quad (395)$$

$$\tilde{w}_{q,\underline{m}_q,3}^u - (\tilde{v}_{q,\underline{m}_q}^u + \tilde{v}_{q,\underline{m}_q}^{u,r}) = v_{q,\underline{m}_q}^u + v_{q,\underline{m}_q}^{u,r} \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_0^q \text{ and } \underline{m}_q = \underline{m}_{q+1} \quad (396)$$

$$\tilde{w}_{q,\underline{m}_q,3}^u - \tilde{v}_{q,\underline{m}_q}^{u,r} = v_{q,\underline{m}_q}^{u,r} \quad \forall q \in B \setminus \{p\} \text{ with } \underline{m}_q \in I_0^q \text{ and } \underline{m}_q \neq \underline{m}_{q+1} \quad (397)$$

$$\tilde{w}_{q,\bar{m}_q,3}^u - (\tilde{v}_{q,\bar{m}_q}^u + \tilde{v}_{q,\bar{m}_q}^{u,l}) = v_{q,\bar{m}_q}^u + v_{q,\bar{m}_q}^{u,l} \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_0^q \text{ and } \bar{m}_q = \bar{m}_{q+1} \quad (398)$$

$$\tilde{w}_{q,\bar{m}_q,3}^u - \tilde{v}_{q,\bar{m}_q}^{u,l} = v_{q,\bar{m}_q}^{u,l} \quad \forall q \in B \setminus \{p\} \text{ with } \bar{m}_q \in I_0^q \text{ and } \bar{m}_q \neq \bar{m}_{q+1} \quad (399)$$

$$\tilde{w}_{pi3}^u - (\tilde{v}_{pi}^{u,l} + \tilde{v}_{pi}^{u,r}) = v_{pi}^{u,l} + v_{pi}^{u,r} \quad \forall i \in I_0^p \setminus \{\underline{m}_p, \bar{m}_p\} \quad (400)$$

$$\tilde{w}_{p,\underline{m}_p,3}^u - \tilde{v}_{p,\underline{m}_p}^{u,r} = v_{p,\underline{m}_p}^{u,r} \quad \text{if } \underline{m}_p \in I_0^p \quad (401)$$

$$\tilde{w}_{p,\bar{m}_p,3}^u - \tilde{v}_{p,\bar{m}_p}^{u,l} = v_{p,\bar{m}_p}^{u,l} \quad \text{if } \bar{m}_p \in I_0^p \quad (402)$$

- Constraints corresponding to vertices $[d, q, i, 1]$

$$\tilde{e}_{1i1}^d + \tilde{t}_{1i}^d - (\tilde{v}_{1i}^{d,l} + \tilde{v}_{1i}^{d,r}) = v_{1i}^{d,l} + v_{1i}^{d,r} \quad \forall i \in I_2^1 \setminus \{\bar{m}_1\} \quad (403)$$

$$\tilde{e}_{1,\bar{m}_1,1}^d + \tilde{t}_{1,\bar{m}_1}^d - \tilde{v}_{1,\bar{m}_1}^{d,l} = v_{1,\bar{m}_1}^{d,l} \quad \text{if } \bar{m}_1 \in I_2^1 \quad (404)$$

$$\tilde{e}_{qi1}^d + \tilde{t}_{qi}^d - (\tilde{v}_{qi}^d + \tilde{v}_{qi}^{d,l} + \tilde{v}_{qi}^{d,r}) = v_{qi}^d + v_{qi}^{d,l} + v_{qi}^{d,r} \quad \forall q \in B \setminus \{1\}, i \in I_2^q \setminus \{\underline{m}_q, \bar{m}_q\} \quad (405)$$

$$\tilde{e}_{q,\underline{m}_q,1}^d + \tilde{t}_{q,\underline{m}_q}^d - (\tilde{v}_{q,\underline{m}_q}^d + \tilde{v}_{q,\underline{m}_q}^{d,l} + \tilde{v}_{q,\underline{m}_q}^{d,r}) = v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,l} + v_{q,\underline{m}_q}^{d,r} \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_2^q \text{ and } \underline{m}_{q-1} < \underline{m}_q \quad (406)$$

$$\tilde{e}_{q,\underline{m}_q,1}^d + \tilde{t}_{q,\underline{m}_q}^d - (\tilde{v}_{q,\underline{m}_q}^d + \tilde{v}_{q,\underline{m}_q}^{d,r}) = v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,r} \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_2^q \text{ and } \underline{m}_{q-1} = \underline{m}_q \quad (407)$$

$$\tilde{e}_{q,\bar{m}_q,1}^d + \tilde{t}_{q,\bar{m}_q}^d - (\tilde{v}_{q,\bar{m}_q}^d + \tilde{v}_{q,\bar{m}_q}^{d,l} + \tilde{v}_{q,\bar{m}_q}^{d,r}) = v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} + v_{q,\bar{m}_q}^{d,r} \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_2^q \text{ and } \bar{m}_{q-1} > \bar{m}_q \quad (408)$$

$$\tilde{e}_{q,\bar{m}_q,1}^d + \tilde{t}_{q,\bar{m}_q}^d - (\tilde{v}_{q,\bar{m}_q}^d + \tilde{v}_{q,\bar{m}_q}^{d,l}) = v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_2^q \text{ and } \bar{m}_{q-1} = \bar{m}_q \quad (409)$$

$$\tilde{e}_{qi1}^d + \tilde{t}_{qi}^d - \tilde{w}_{qi1}^d = w_{qi1}^d \quad \forall q \in B, i \in I^q \setminus (I_1^q \cup I_2^q) \quad (410)$$

- Constraints corresponding to vertices $[d, q, i, 2]$

$$\tilde{e}_{1i2}^d + \tilde{w}_{1i1}^d - (\tilde{v}_{1i}^{d,l} + \tilde{v}_{1i}^{d,r}) = v_{1i}^{d,l} + v_{1i}^{d,r} \quad \forall i \in I_3^1 \setminus \{\bar{m}_1\} \quad (411)$$

$$\tilde{e}_{1,\bar{m}_1,2}^d + \tilde{w}_{1,\bar{m}_1,1}^d - \tilde{v}_{1,\bar{m}_1}^{d,l} = v_{1,\bar{m}_1}^{d,l} \quad \text{if } \bar{m}_1 \in I_3^1 \quad (412)$$

$$\tilde{e}_{qi2}^d + \tilde{w}_{qi1}^d - (\tilde{v}_{qi}^d + \tilde{v}_{qi}^{d,l} + \tilde{v}_{qi}^{d,r}) = v_{qi}^d + v_{qi}^{d,l} + v_{qi}^{d,r} \quad \forall q \in B \setminus \{1\}, i \in I_3^q \setminus \{\underline{m}_q, \bar{m}_q\} \quad (413)$$

$$\tilde{e}_{q,\underline{m}_q,2}^d + \tilde{w}_{q,\underline{m}_q,1}^d - (\tilde{v}_{q,\underline{m}_q}^d + \tilde{v}_{q,\underline{m}_q}^{d,l} + \tilde{v}_{q,\underline{m}_q}^{d,r}) = v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,l} + v_{q,\underline{m}_q}^{d,r} \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_3^q \text{ and } \underline{m}_{q-1} < \underline{m}_q \quad (414)$$

$$\tilde{e}_{q,\underline{m}_q,2}^d + \tilde{w}_{q,\underline{m}_q,1}^d - \tilde{v}_{q,\underline{m}_q}^d + \tilde{v}_{q,\underline{m}_q}^{d,r} = v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,r} \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_3^q \text{ and } \underline{m}_{q-1} = \underline{m}_q \quad (415)$$

$$\tilde{e}_{q,\bar{m}_q,2}^d + \tilde{w}_{q,\bar{m}_q,1}^d - (\tilde{v}_{q,\bar{m}_q}^d + \tilde{v}_{q,\bar{m}_q}^{d,l} + \tilde{v}_{q,\bar{m}_q}^{d,r}) = v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} + v_{q,\bar{m}_q}^{d,r} \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_3^q \text{ and } \bar{m}_{q-1} > \bar{m}_q \quad (416)$$

$$\tilde{e}_{q,\bar{m}_q,2}^d + \tilde{w}_{q,\bar{m}_q,1}^d - (\tilde{v}_{q,\bar{m}_q}^d + \tilde{v}_{q,\bar{m}_q}^{d,l}) = v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_3^q \text{ and } \bar{m}_{q-1} = \bar{m}_q \quad (417)$$

$$\tilde{w}_{qi1}^d - \tilde{w}_{qi2}^d = w_{qi2}^d \quad \forall q \in B, i \in I_0^q \cup I_4^q \quad (418)$$

$$\tilde{e}_{qi2}^d + \tilde{w}_{qi1}^d - \tilde{w}_{qi2}^d = w_{qi2}^d \quad \forall q \in B, i \in I_5^q \quad (419)$$

- Constraints corresponding to vertices $[d, q, i, 3]$

$$\tilde{e}_{1i3}^d + \tilde{w}_{1i2}^d - (\tilde{v}_{1i}^{d,l} + \tilde{v}_{1i}^{d,r}) = v_{1i}^{d,l} + v_{1i}^{d,r} \quad \forall i \in I_4^1 \setminus \{\bar{m}_1\} \quad (420)$$

$$\tilde{e}_{1,\bar{m}_1,3}^d + \tilde{w}_{1,\bar{m}_1,2}^d - \tilde{v}_{1,\bar{m}_1}^{d,l} = v_{1,\bar{m}_1}^{d,l} \quad \text{if } \bar{m}_1 \in I_4^1 \quad (421)$$

$$\tilde{e}_{qi3}^d + \tilde{w}_{qi2}^d - (\tilde{v}_{qi}^d + \tilde{v}_{qi}^{d,l} + \tilde{v}_{qi}^{d,r}) = v_{qi}^d + v_{qi}^{d,l} + v_{qi}^{d,r} \quad \forall q \in B \setminus \{1\}, i \in I_4^q \setminus \{\underline{m}_q, \bar{m}_q\} \quad (422)$$

$$\tilde{e}_{q,\underline{m}_q,3}^d + \tilde{w}_{q,\underline{m}_q,2}^d - (\tilde{v}_{q,\underline{m}_q}^d + \tilde{v}_{q,\underline{m}_q}^{d,l} + \tilde{v}_{q,\underline{m}_q}^{d,r}) = v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,l} + v_{q,\underline{m}_q}^{d,r} \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_4^q \text{ and } \underline{m}_{q-1} < \underline{m}_q \quad (423)$$

$$\tilde{e}_{q,\underline{m}_q,3}^d + \tilde{w}_{q,\underline{m}_q,2}^d - \tilde{v}_{q,\underline{m}_q}^d + \tilde{v}_{q,\underline{m}_q}^{d,r} = v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,r} \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_4^q \text{ and } \underline{m}_{q-1} = \underline{m}_q \quad (424)$$

$$\tilde{e}_{q,\bar{m}_q,3}^d + \tilde{w}_{q,\bar{m}_q,2}^d - (\tilde{v}_{q,\bar{m}_q}^d + \tilde{v}_{q,\bar{m}_q}^{d,l} + \tilde{v}_{q,\bar{m}_q}^{d,r}) = v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} + v_{q,\bar{m}_q}^{d,r} \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_4^q \text{ and } \bar{m}_{q-1} > \bar{m}_q \quad (425)$$

$$\tilde{e}_{q,\bar{m}_q,3}^d + \tilde{w}_{q,\bar{m}_q,2}^d - (\tilde{v}_{q,\bar{m}_q}^d + \tilde{v}_{q,\bar{m}_q}^{d,l}) = v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_4^q \text{ and } \bar{m}_{q-1} = \bar{m}_q \quad (426)$$

$$\tilde{w}_{1i2}^d - (\tilde{v}_{1i}^{d,l} + \tilde{v}_{1i}^{d,r}) = v_{1i}^{d,l} + v_{1i}^{d,r} \quad \forall i \in I_5^1 \setminus \{\bar{m}_1\} \quad (427)$$

$$\tilde{w}_{1,\bar{m}_1,2}^d - \tilde{v}_{1,\bar{m}_1}^{d,l} = v_{1,\bar{m}_1}^{d,l} \quad \text{if } \bar{m}_1 \in I_5^1 \quad (428)$$

$$\tilde{w}_{qi2}^d - (\tilde{v}_{qi}^d + \tilde{v}_{qi}^{d,l} + \tilde{v}_{qi}^{d,r}) = v_{qi}^d + v_{qi}^{d,l} + v_{qi}^{d,r} \quad \forall q \in B \setminus \{1\}, i \in I_5^q \setminus \{\underline{m}_q, \bar{m}_q\} \quad (429)$$

$$\tilde{w}_{q,\underline{m}_q,2}^d - (\tilde{v}_{q,\underline{m}_q}^d + \tilde{v}_{q,\underline{m}_q}^{d,l} + \tilde{v}_{q,\underline{m}_q}^{d,r}) = v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,l} + v_{q,\underline{m}_q}^{d,r} \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_5^q \text{ and } \underline{m}_{q-1} < \underline{m}_q \quad (430)$$

$$\tilde{w}_{q,\underline{m}_q,2}^d - (\tilde{v}_{q,\underline{m}_q}^d + \tilde{v}_{q,\underline{m}_q}^{d,r}) = v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,r} \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_5^q \text{ and } \underline{m}_{q-1} = \underline{m}_q \quad (431)$$

$$\tilde{w}_{q,\bar{m}_q,2}^d - (\tilde{v}_{q,\bar{m}_q}^d + \tilde{v}_{q,\bar{m}_q}^{d,l} + \tilde{v}_{q,\bar{m}_q}^{d,r}) = v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} + v_{q,\bar{m}_q}^{d,r} \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_5^q \text{ and } \bar{m}_{q-1} > \bar{m}_q \quad (432)$$

$$\tilde{w}_{q,\bar{m}_q,2}^d - (\tilde{v}_{q,\bar{m}_q}^d + \tilde{v}_{q,\bar{m}_q}^{d,l}) = v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_5^q \text{ and } \bar{m}_{q-1} = \bar{m}_q \quad (433)$$

$$\tilde{e}_{qi3}^d + \tilde{w}_{qi2}^d - \tilde{w}_{qi3}^d = w_{qi3}^d \quad \forall q \in B, i \in I_0^q \quad (434)$$

- Constraints corresponding to vertices $[d, q, i, 4]$

$$\tilde{w}_{1i3}^d - (\tilde{v}_{1i}^{d,l} + \tilde{v}_{1i}^{d,r}) = v_{1i}^{d,l} + v_{1i}^{d,r} \quad \forall i \in I_0^1 \setminus \{\bar{m}_1\} \quad (435)$$

$$\tilde{w}_{1,\bar{m}_1,3}^d - \tilde{v}_{1,\bar{m}_1}^{d,l} = v_{1,\bar{m}_1}^{d,l} \quad \text{if } \bar{m}_1 \in I_0^1 \quad (436)$$

$$\tilde{w}_{qi3}^d - (\tilde{v}_{qi}^d + \tilde{v}_{qi}^{d,l} + \tilde{v}_{qi}^{d,r}) = v_{qi}^d + v_{qi}^{d,l} + v_{qi}^{d,r} \quad \forall q \in B \setminus \{1\}, i \in I_0^q \setminus \{\underline{m}_q, \bar{m}_q\} \quad (437)$$

$$\tilde{w}_{q,\underline{m}_q,3}^d - (\tilde{v}_{q,\underline{m}_q}^d + \tilde{v}_{q,\underline{m}_q}^{d,l} + \tilde{v}_{q,\underline{m}_q}^{d,r}) = v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,l} + v_{q,\underline{m}_q}^{d,r} \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_0^q \text{ and } \underline{m}_{q-1} < \underline{m}_q \quad (438)$$

$$\tilde{w}_{q,\underline{m}_q,3}^d - (\tilde{v}_{q,\underline{m}_q}^d + \tilde{v}_{q,\underline{m}_q}^{d,r}) = v_{q,\underline{m}_q}^d + v_{q,\underline{m}_q}^{d,r} \quad \forall q \in B \setminus \{1\} \text{ with } \underline{m}_q \in I_0^q \text{ and } \underline{m}_{q-1} = \underline{m}_q \quad (439)$$

$$\tilde{w}_{q,\bar{m}_q,3}^d - (\tilde{v}_{q,\bar{m}_q}^d + \tilde{v}_{q,\bar{m}_q}^{d,l} + \tilde{v}_{q,\bar{m}_q}^{d,r}) = v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} + v_{q,\bar{m}_q}^{d,r} \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_0^q \text{ and } \bar{m}_{q-1} > \bar{m}_q \quad (440)$$

$$\tilde{w}_{q,\bar{m}_q,3}^d - (\tilde{v}_{q,\bar{m}_q}^d + \tilde{v}_{q,\bar{m}_q}^{d,l}) = v_{q,\bar{m}_q}^d + v_{q,\bar{m}_q}^{d,l} \quad \forall q \in B \setminus \{1\} \text{ with } \bar{m}_q \in I_0^q \text{ and } \bar{m}_{q-1} = \bar{m}_q \quad (441)$$

- Constraints to link variables

$$\tilde{r}_{qi}^r \leq M \cdot r_{qi}^r \quad \forall (q, i) \in F \times (I^q \setminus \{\bar{m}^q - 1, \bar{m}^q\}) \quad (442)$$

$$\tilde{r}_{qi}^d \leq M \cdot r_{qi}^d \quad \forall (q, i) \in (F \setminus \{1\}) \times (I^q \setminus \{\bar{m}^q\}) \quad (443)$$

$$\tilde{r}_{qi}^u \leq M \cdot r_{qi}^u \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus \{\bar{m}^q\}) \quad (444)$$

$$\tilde{l}_{qi}^l \leq M \cdot l_{qi}^l \quad \forall (q, i) \in F \times ((I^q \setminus \{\underline{m}^q, \bar{m}^q + 1\}) \cup \{(1, 2)\}) \quad (445)$$

$$\tilde{l}_{qi}^d \leq M \cdot l_{qi}^d \quad \forall (q, i) \in (F \setminus \{1\}) \times (I^q \setminus \{\underline{m}^q\}) \quad (446)$$

$$\tilde{l}_{qi}^u \leq M \cdot l_{qi}^u \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus \{\underline{m}^q\}) \quad (447)$$

$$\tilde{e}_{qi1}^u \leq M \cdot e_{qi1}^u \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q) \quad (448)$$

$$\tilde{e}_{qi1}^d \leq M \cdot e_{qi1}^d \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q) \quad (449)$$

$$\tilde{e}_{qi2}^u \leq M \cdot e_{qi2}^u \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I_3^q \cup I_4^q) \quad (450)$$

$$\tilde{e}_{qi2}^d \leq M \cdot e_{qi2}^d \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I_3^q \cup I_5^q) \quad (451)$$

$$\tilde{e}_{qi3}^u \leq M \cdot e_{qi3}^u \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I_0^q \cup I_5^q) \quad (452)$$

$$\tilde{e}_{qi3}^d \leq M \cdot e_{qi3}^d \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I_0^q \cup I_4^q) \quad (453)$$

$$\tilde{t}_{qi}^u \leq M \cdot t_{qi}^u \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q) \quad (454)$$

$$\tilde{t}_{qi}^d \leq M \cdot t_{qi}^d \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q) \quad (455)$$

$$\tilde{w}_{qis}^u \leq M \cdot w_{qis}^u \quad \forall (q, i, s) \in B \times ((I_3^q \times \{1\}) \cup ((I_4^q \cup I_5^q) \times \{1, 2\}) \cup (I_0^q \times \{1, 2, 3\})) \quad (456)$$

$$\tilde{w}_{qis}^d \leq M \cdot w_{qis}^d \quad \forall (q, i, s) \in B \times ((I_3^q \times \{1\}) \cup ((I_4^q \cup I_5^q) \times \{1, 2\}) \cup (I_0^q \times \{1, 2, 3\})) \quad (457)$$

$$\tilde{v}_{qi}^u \leq M \cdot v_{qi}^u \quad \forall (q, i) \in (B \setminus \{p\}) \times I^q \quad (458)$$

$$\tilde{v}_{qi}^{u,l} \leq M \cdot v_{qi}^{u,l} \quad \forall (q, i) \in B \times (I^q \setminus \{\underline{m}_q\}) \quad (459)$$

$$\tilde{v}_{qi}^{u,r} \leq M \cdot v_{qi}^{u,r} \quad \forall (q, i) \in B \times (I^q \setminus \{\overline{m}_q\}) \quad (460)$$

$$\tilde{v}_{qi}^d \leq M \cdot v_{qi}^d \quad \forall (q, i) \in (B \setminus \{1\}) \times I^q \quad (461)$$

$$\tilde{v}_{qi}^{d,l} \leq M \cdot v_{qi}^{d,l} \quad \forall (q, i) \in B \times ((I^q \setminus \{\underline{m}_q\}) \cup \{(1, 1)\}) \quad (462)$$

$$\tilde{v}_{qi}^{d,r} \leq M \cdot v_{qi}^{d,r} \quad \forall (q, i) \in B \times (I^q \setminus \{\overline{m}_q\}) \quad (463)$$

$$\tilde{y}_\alpha^0 \leq M \cdot y_\alpha^0 \quad \forall \alpha \in \{l, r, u\} \quad (464)$$

$$\tilde{y}_0^l \leq M \cdot y_0^l \quad (465)$$

Constraints for the Domains of the Variables:

$$r_{qi}^r \in \{0, 1\} \quad \forall (q, i) \in F \times (I^q \setminus \{\overline{m}^q - 1, \overline{m}^q\}) \quad (466)$$

$$r_{qi}^d \in \{0, 1\} \quad \forall (q, i) \in (F \setminus \{1\}) \times (I^q \setminus \{\overline{m}^q\}) \quad (467)$$

$$r_{qi}^u \in \{0, 1\} \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus \{\overline{m}^q\}) \quad (468)$$

$$l_{qi}^l \in \{0, 1\} \quad \forall (q, i) \in F \times ((I^q \setminus \{\underline{m}^q, \overline{m}^q + 1\}) \cup \{(1, 2)\}) \quad (469)$$

$$l_{qi}^d \in \{0, 1\} \quad \forall (q, i) \in (F \setminus \{1\}) \times (I^q \setminus \{\underline{m}^q\}) \quad (470)$$

$$l_{qi}^u \in \{0, 1\} \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus \{\underline{m}^q\}) \quad (471)$$

$$e_{qi1}^u \in \{0, 1\} \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q) \quad (472)$$

$$e_{qi1}^d \in \{0, 1\} \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q) \quad (473)$$

$$e_{qi2}^u \in \{0, 1\} \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I_3^q \cup I_4^q) \quad (474)$$

$$e_{qi2}^d \in \{0, 1\} \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I_3^q \cup I_5^q) \quad (475)$$

$$e_{qi3}^u \in \{0, 1\} \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I_0^q \cup I_5^q) \quad (476)$$

$$e_{qi3}^d \in \{0, 1\} \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I_0^q \cup I_4^q) \quad (477)$$

$$t_{qi}^u \in \{0, 1\} \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q) \quad (478)$$

$$t_{qi}^d \in \{0, 1\} \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q) \quad (479)$$

$$w_{qis}^u \in \{0, 1\} \quad \forall (q, i, s) \in B \times ((I_3^q \times \{1\}) \cup ((I_4^q \cup I_5^q) \times \{1, 2\}) \cup (I_0^q \times \{1, 2, 3\})) \quad (480)$$

$$w_{qis}^d \in \{0, 1\} \quad \forall (q, i, s) \in B \times ((I_3^q \times \{1\}) \cup ((I_4^q \cup I_5^q) \times \{1, 2\}) \cup (I_0^q \times \{1, 2, 3\})) \quad (481)$$

$$v_{qi}^u \in \{0, 1\} \quad \forall (q, i) \in (B \setminus \{p\}) \times I^q \quad (482)$$

$$v_{qi}^{u,l} \in \{0, 1\} \quad \forall (q, i) \in B \times (I^q \setminus \{\underline{m}_q\}) \quad (483)$$

$$v_{qi}^{u,r} \in \{0, 1\} \quad \forall (q, i) \in B \times (I^q \setminus \{\bar{m}_q\}) \quad (484)$$

$$v_{qi}^d \in \{0, 1\} \quad \forall (q, i) \in (B \setminus \{1\}) \times I^q \quad (485)$$

$$v_{qi}^{d,l} \in \{0, 1\} \quad \forall (q, i) \in B \times ((I^q \setminus \{\underline{m}_q\}) \cup \{(1, 1)\}) \quad (486)$$

$$v_{qi}^{d,r} \in \{0, 1\} \quad \forall (q, i) \in B \times (I^q \setminus \{\bar{m}_q\}) \quad (487)$$

$$y_\alpha^0 \in \{0, 1\} \quad \forall \alpha \in \{l, r, u\} \quad (488)$$

$$y_0^l \in \{0, 1\} \quad (489)$$

$$\tilde{r}_{qi}^r \geq 0 \quad \forall (q, i) \in F \times (I^q \setminus \{\bar{m}^q - 1, \bar{m}^q\}) \quad (490)$$

$$\tilde{r}_{qi}^d \geq 0 \quad \forall (q, i) \in (F \setminus \{1\}) \times (I^q \setminus \{\bar{m}^q\}) \quad (491)$$

$$\tilde{r}_{qi}^u \geq 0 \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus \{\bar{m}^q\}) \quad (492)$$

$$\tilde{l}_{qi}^l \geq 0 \quad \forall (q, i) \in F \times ((I^q \setminus \{\underline{m}^q, \underline{m}^q + 1\}) \cup \{(1, 2)\}) \quad (493)$$

$$\tilde{l}_{qi}^d \geq 0 \quad \forall (q, i) \in (F \setminus \{1\}) \times (I^q \setminus \{\underline{m}^q\}) \quad (494)$$

$$\tilde{l}_{qi}^u \geq 0 \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus \{\underline{m}^q\}) \quad (495)$$

$$\tilde{e}_{qi1}^u \geq 0 \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q) \quad (496)$$

$$\tilde{e}_{qi1}^d \geq 0 \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q) \quad (497)$$

$$\tilde{e}_{qi2}^u \geq 0 \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I_3^q \cup I_4^q) \quad (498)$$

$$\tilde{e}_{qi2}^d \geq 0 \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I_3^q \cup I_5^q) \quad (499)$$

$$\tilde{e}_{qi3}^u \geq 0 \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I_0^q \cup I_5^q) \quad (500)$$

$$\tilde{e}_{qi3}^d \geq 0 \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I_0^q \cup I_4^q) \quad (501)$$

$$\tilde{l}_{qi}^u \geq 0 \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q) \quad (502)$$

$$\tilde{l}_{qi}^d \geq 0 \quad \forall (q, i) \in (F \setminus \{p+1\}) \times (I^q \setminus I_1^q) \quad (503)$$

$$\tilde{w}_{qis}^u \geq 0 \quad \forall (q, i, s) \in B \times ((I_3^q \times \{1\}) \cup ((I_4^q \cup I_5^q) \times \{1, 2\}) \cup (I_0^q \times \{1, 2, 3\})) \quad (504)$$

$$\tilde{w}_{qis}^d \geq 0 \quad \forall (q, i, s) \in B \times ((I_3^q \times \{1\}) \cup ((I_4^q \cup I_5^q) \times \{1, 2\}) \cup (I_0^q \times \{1, 2, 3\})) \quad (505)$$

$$\tilde{v}_{qi}^u \geq 0 \quad \forall (q, i) \in (B \setminus \{p\}) \times I^q \quad (506)$$

$$\tilde{v}_{qi}^{u,l} \geq 0 \quad \forall (q, i) \in B \times (I^q \setminus \{\underline{m}_q\}) \quad (507)$$

$$\tilde{v}_{qi}^{u,r} \geq 0 \quad \forall (q, i) \in B \times (I^q \setminus \{\bar{m}_q\}) \quad (508)$$

$$\tilde{v}_{qi}^d \geq 0 \quad \forall (q, i) \in (B \setminus \{1\}) \times I^q \quad (509)$$

$$\tilde{v}_{qi}^{d,l} \geq 0 \quad \forall (q, i) \in B \times ((I^q \setminus \{\underline{m}_q\}) \cup \{(1, 1)\}) \quad (510)$$

$$\tilde{v}_{qi}^{d,r} \geq 0 \quad \forall (q, i) \in B \times (I^q \setminus \{\bar{m}_q\}) \quad (511)$$

$$\tilde{y}_\alpha^0 \geq 0 \quad \forall \alpha \in \{l, r, u\} \quad (512)$$

$$\tilde{y}_0^l \geq 0 \quad (513)$$

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